Voluntary Contributions Vote out Public Ones*

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Abstract

Many public goods are supported with both private and public funding. It is often argued that public funding, based on taxes, crowds out private philanthropic contributions. Agents respond to increases in taxes by decreasing their donations. However, the tax level depends on the political equilibrium arising from agents’ voting decisions. In this paper we analyze a variety of motivations for voluntary donations and show that when philanthropic preferences become more prevalent in the society, the equilibrium tax level will tend to be lower and, more surprisingly, the stock of the public good is likely to decrease.

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1 Introduction

Many public goods (such as culture, art, public works or resources to the poor) are supported with both private and public funding. On the one hand, public funding is typically carried out using distortionary taxation, imposed to all agents in the society. On the other hand, private provision is the result of the donations of individual agents or institutions that may not benefit directly from their transfer. In the U.S., for instance, the majority of philanthropical donations are from individuals (76% in data from year 2002), while the rest is provided by foundations, corporations, etc. Over the last 30 years, individual donations amounted to around 2% of income. The declining trend in these donations has been reversed in recent years. At an international level, data is scarce; however, overall private donations also amount to a significative share of the financing of some of the public goods.¹

One of the key issues analyzed by the literature on philanthropy is the extent to which the (exogenous) public funding crowds-out the private provision to a public good. Acknowledging that public and private contributions are not chosen independently, most papers have emphasized the implications that changes in an exogenous tax rate have on the voluntary contributions made by individuals. They show that, under a variety of assumptions, taxes might crowd out private funding. In this paper, however, we show that when both private and public funding are endogeneized (in a political-economy framework), the crowding-out result changes since voluntary contributions also feed back into the equilibrium tax rate. Voters in more philanthropic societies tend to choose lower taxes that entail lower public funding. In a nutshell, voluntary contributions (arising from a variety of motivations) might vote out public ones. More strikingly, the equilibrium level of the public good might decrease as a result.

The private provision of public goods has long puzzled economists. The motivation put forward by pioneer works in the literature such as Bergstrom et al. (1986) and Warr (1982) is self-interest in the level of the public good. Because agents only internalize their private return from the public good, provision will in general be inefficiently low. However, an important result is that taxes (and the corresponding public expenditure) is not likely to solve this problem, because they are seen by agents as a substitute of their private contributions. That is, increases in the tax rate will give rise to a one-to-one decrease in private philanthropy. As a result, increases in the tax rate are only likely to raise the stock of the public good for

¹See Andreoni (2004) for a recent review of the theoretical and empirical literature on philanthropy.
those agents that do not make any voluntary contribution. Moreover, because taxation is
distortionary, one could regard taxation as a bad substitute of private contributions. 2

Andreoni (1990) notes that this “classical” motivation for voluntary contributions lacks
predictive power for several reasons. First, empirical results show that crowding out is
likely to be small. And second, as Andreoni (1988) shows, in large communities, the impact
of individual contributions on the level of the public good is limited, challenging the self-
interest motivation as a candidate to explain voluntary contributions. Andreoni proposes a
natural way to reconcile theory and empirical evidence under the name of warm-glow giving
preferences. It is assumed that agents prefer to support the public good through voluntary
contributions (philanthropy) rather than through levied taxes. As a consequence of such
preferences, even when public funding increases, agents would not reduce their contributions
in the same amount since they derive direct utility from the act of giving voluntarily.

In this paper we consider an economy in which the public good is jointly financed by public
and private contributions. We abstract from the self-interest motivation by considering a
continuum of agents and, instead, we explain private contributions by assuming that agents
have heterogeneous warm-glow giving preferences. Then, for a given tax level, more intense
warm-glow giving results in larger voluntary contributions. As a result, more philanthropic
societies enjoy a larger stock of the public good. Moreover, comparative-statics analysis
related to the tax rate delivers the same (partial) crowding out studied in the literature.

This rather conventional model of warm-glow giving is then embedded in a previous stage
where agents collectively decide on the tax level in the economy. Different intensities in warm-
glow giving among agents translate into different preferences over the tax rate. We study
how the distribution of warm-glow giving preferences aggregates into the equilibrium tax
rate and the stock of the public good. The main result of the paper is that a higher level of
philanthropy results in a lower level of public funding and, as a consequence, may yield a lower
level of public good provision. The intuition of this result is rather straightforward. Agents,
when collectively deciding on the level of public funding for the good, take into account
the warm-glow giving preferences of all voters. As a result, in more “altruistic” societies
(understood as societies in which agents have stronger warm-glow giving preferences), agents
have a double incentive to choose lower tax levels. First, since society as a whole is more

2Another strong theoretical result of this literature is that small redistributions of income among givers
are neutral to the total supply of the public good, or on individual consumption. This would be due to the
fact that people give to a common public good, which would undo the effect of the redistribution (see Warr
(1983)). Along the same lines, Bernheim (1986) provides several additional neutrality results.
altruistic, taxes are less necessary to obtain contributions by the other agents: they choose to do so on their own. This is what we denote as the aggregate effect. And second, since a more altruistic agent (stronger warm-glow giving) derives a higher utility from voluntary (rather than mandatory) contributions, he also prefers lower taxes. This is what we denote as the median-voter effect.

Then, one consequence of such (reverse) crowding-out is that the stock of the public good will not necessarily be monotonic in the level of altruism in the society. In particular, in our model, non-altruistic societies elect the tax level that yields the efficient level of the public good. However, societies in which agents have stronger warm-glow giving preferences (and make larger voluntary contributions) may actually end up with a lower level of public good provision.

These results are in stark contrast with the standard implications obtained in the literature on philanthropy, in which public funding crowds-out voluntary contributions (imperfectly, in the presence of warm-glow giving). While recommendations in that case play down the role of taxes as a way to finance the public good, our results suggest that lower equilibrium taxes, even accounting for the larger equilibrium private contributions generated, may signal additional inefficiencies.

The mechanism behind the inefficiencies in the collective choice is not specific of the warm-glow giving preferences. We show that, to a large extent, the same result holds for a larger family of preferences. As an example, we show that crowding-out will occur even if agents do not obtain any utility from donations per se but, rather, they are motivated by the relative size of their donation with respect to average donation in the society as a whole. These preferences are related to the so-called prosocial behavior studied by Benabou and Tirole (2006). This example also illustrates that although the median-voter effect might not be general, the aggregate effect is robust to changes in preferences. This sole effect is enough for the stock of the public good not to be monotonic in the level of philanthropy present in the society.

Our model builds on the extensive literature that studies the collective decision regarding the provision of a public good, started with papers such as Meltzer and Richard (1981). Heterogeneous agents (in income or labor productivity, for example) decide through majority voting the provision of a public good or the level of redistribution existing in society. However, agents anticipate in their voting decision the effect that each policy has not only on them but on the rest of the society as well. In this paper we study the interplay between the tax
level implemented in a society and the level of the provision of the public good when agents are heterogeneous in their warm-glow giving.

To the best of our knowledge, ours and Epple and Romano (2003) are the only papers to endogeneize both sources of funding which, thus, makes Epple and Romano (2003) a clear benchmark to our analysis. They endogeneize the collective decision over the public funding of a public good in the presence of voluntary contributions, but absent any warm-glow giving utility. They show that, while dual provision (public and voluntary contributions) arises in equilibrium in small economies, dual provision will not actually materialize in a sufficiently large economy. Our analysis differs from that paper in two respects. First, by focusing in a continuum of agents, voluntary contributions are only driven by the warm-glow giving preferences. Second, whereas in their model heterogeneous political preferences are due to heterogeneous income, in our model agents differ only in the intensity of their warm-glow giving.

The rest of the paper is organized as follows. In the next section we outline the benchmark model and present the main results of our paper. Then, in section 3, with alternative preference specifications, we show that the analysis and main results of section 2 still apply. Section 4 then concludes. Proofs are relegated to the appendix.

2 The Model

Consider an economy with a continuum of agents of mass 1. An agent’s utility depends on the stock of a public good, denoted as $G$, together with his individual contribution to this public good, $g$, and the tax $\tau$. In particular, we model an agent’s utility as

$$ U(g, \tau) = G + \theta (g - \tau) - C(g). $$

This utility function assumes that the contribution to the public good has an opportunity cost $C(g)$ which is increasing and convex. For simplicity, we normalize this cost so that $C'(1) = 1$. The tax establishes a minimum level for the contribution to the public good, so that

$$ g \geq \tau. $$

Thus, while $\tau$ is the tax levied to contribute to the public good (through the public sector), $g - \tau$ is the voluntary contribution of the agent to the provision of the public good. In this economy, agents derive utility from the act of contributing to the public good, but only from
the voluntary part (not from the taxes paid). This assumption is known in the literature on philanthropy as having warm-glow giving preferences. The intensity of this warm-glow giving is summarized in the parameter $\theta \in [0, 1]$. Agents are heterogeneous in this parameter according to the distribution function $\Phi(\theta, s)$ where $s$ orders the distribution in the first order stochastic sense. Warm-glow preferences are usually denoted as preferences with imperfect altruism. For this reason, and with some abuse of terminology, we will denote a distribution with a higher $s$ as one pertaining to a more altruistic society.\footnote{Notice that this concept does not correspond to the usual meaning of altruism, where the utility of others is embedded in the own utility function.}

Finally, notice that due to the assumption that there is a continuum of agents, the effect of $g$ on utility occurs only through the warm-glow giving of the agents, since each one has a negligible impact over the stock of the public good.\footnote{In a more general framework where each agent has a non-negligible impact on the stock of the public good, Ribar and Wilhelm (2002)) shows that as the economy grows large, warm-glow will become the dominant, if not the exclusive motive for giving at the margin.}

We consider the decisions of agents in two stages. In the first stage agents simultaneously decide (through majority voting) on the level of the tax level $\tau$. In the second stage agents choose independently the level of their contribution, $g$, constrained to the condition $g \geq \tau$. We solve the model by backward induction. Before, though, we characterize the first best level of contribution to the public good.

### 2.1 The First Best

An important preliminary question in the characterization of the first best is whether the utility derived directly from the act of giving should be included in the aggregate utility considered, for instance, by a social planner. Andreoni (2004) provides an extensive discussion on this subject concluding that it should be left out. As a result, the value of $g$ that maximizes welfare, understood as the sum of the utility of all consumers (leaving out the warm-glow from giving), solves

$$\max_{g} G - C(g),$$

which given our assumption on $C$ implies $g^* = 1$. 

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Notice that this concept does not correspond to the usual meaning of altruism, where the utility of others is embedded in the own utility function.
2.2 The Second Stage: Contributing

We start by characterizing the utility-maximizing contribution in the second stage for a given level of $\tau$. Agents choose $g$ so as to

$$\max_{g \geq \tau} G + \theta (g - \tau) - C (g).$$

Since, as we said above, each agent has a zero impact on $G$, only the intrinsic utility from giving drives the choice of $g$. This leads to a first order condition (in the case in which $g$ is not binding) of

$$\theta = C' (\hat{g}),$$

which from the assumptions on $C$ implies that $\hat{g} (\theta)$ is increasing in $\theta$. Therefore, the privately optimal contribution to the public good corresponds to

$$g^* (\theta) = \max \{ \hat{g} (\theta), \tau \}. \quad (1)$$

After aggregating among all agents, the total contribution in the society can be computed as

$$G \equiv \int g^* (\theta) \phi (\theta, s) \, d\theta.$$

Remark 1 For a given level of taxation $\tau$, a more altruistic society will yield a higher stock of the public good. In other words, for a given $\tau$, $G$ is increasing in $s$.

Remark 2 At the individual level, an increase in the tax rate crowds out voluntary contributions at a one-to-one rate. However, at the aggregate level, and consistent with the literature on philanthropy, crowding out is not perfect, since only constrained agents increase their overall contribution. Hence, for a given distribution of preferences, a higher tax rate yields a higher level of the public good (whenever the tax rate is binding for some agents).

2.3 The First Stage: Voting

We now turn to the first stage of the game in which agents collectively determine the tax through majority voting. Anticipating their own voluntary contributions and those of the rest of the agents, the utility of voters in the collective decision game can be described as a function of the tax, $\tau$, their level of prosocial behavior, $\theta$, and its distribution over the society, $s$. That is,

$$V (\theta, \tau, s) \equiv \int g^* (x) \phi (x, s) \, dx + \theta (g^* (\theta) - \tau) - C (g^* (\theta))$$

$$= \Phi (\bar{\theta} (\tau), s) \tau + \int_{\theta (\tau)} \hat{g} (x) \phi (x, s) \, dx + \theta (g^* (\theta) - \tau) - C (g^* (\theta)). \quad (2)$$
In the last expression, we have implicitly defined $\theta(\tau)$ as the highest value of $\theta$ for which the choice of the private contribution is constrained by the tax level. In particular, 

$$\hat{g}(\theta) = \tau,$$

(3)

which implies that $\theta$ is increasing in $\tau$ due to the monotonicity of $\hat{g}$.

We denote by $\tau^*(\theta, s)$ the tax level that maximizes the utility of an agent of type $\theta$. In other words,

$$\tau^* \in \arg \max V(\theta, \tau, s).$$

The next two lemmas state two important properties of the function $V$.

**Lemma 1** The function $V$ is submodular in $s$ and $\tau$.

The optimal choice of $\tau$ of a given individual trade-offs an individual effect over the own contribution and a social effect that corresponds to the total contribution made by others. On the one hand, for an agent it is always individually costly to increase $\tau$. If the agent’s decision is constrained by $\tau$, an increase in the tax entails a higher cost of the contribution. If instead the agent’s contribution is not constrained, increasing $\tau$ reduces his warm-glow giving.

On the other hand, an increase in $\tau$ has the positive effect of increasing the sum of all contributions. As a consequence, if the distribution of the population becomes more altruistic (a higher $s$), fewer people will be constrained by the same tax, and the positive social effect of increasing $\tau$ will be reduced (since many people will be contributing anyway). Hence, the larger is $s$, the less willing is any given agent to increase $\tau$.\textsuperscript{5} We will denote this result as the aggregate effect.

A second property of the function $V$ behind the results in this paper is stated next.

**Lemma 2** The function $V$ is submodular in $\theta$ and $\tau$.

Any given agent is aware that a higher tax implies a higher level of the public good $G$. However, for a more altruistic agent (one with a higher $\theta$), a higher tax makes a voluntary contribution beyond the tax level more expensive. For this reason, he is interested in setting a lower tax in order to benefit from his additional voluntary contribution.

\textsuperscript{5}Notice that our preference specification assumes that the value of the marginal contribution to the public good is the same regardless of its level. Adding decreasing returns to this contribution would reinforce our results.
Since society is heterogeneous exclusively in the intensity of warm-glow giving preferences, monotonicity in the choice of $\tau$ is enough to guarantee that the result of majority voting corresponds to the preferred choice of the median voter. We refer to this median voter as $\hat{\theta}$.

Consider two distributions of warm-glow giving with $s' > s$. According to the aggregate effect (lemma 1), if we compare two identical voters in the two distributions, the one that corresponds to a more altruistic society will prefer a lower tax rate. Moreover, under the distribution $s'$ the median voter will have a more intense warm-glow giving preference. Then, from the result of lemma 2 we can also conclude that this more altruistic median voter will prefer a lower tax level (we denote this result as the median-voter effect). Hence, this analysis allows us to state the following proposition.

**Proposition 3**  *Majority voting in a more altruistic society leads to a lower tax rate.*

We proceed next to compute specifically the equilibrium tax rate. The utility function turns out to be non-concave in $\tau$. However, it is easy to see that two cases need to be considered depending on whether a voter of type $\theta$ chooses a contribution beyond his unconstrained choice $\hat{g}(\theta)$ or not. In the second case, with $g^* = \tau$, the first order condition for the voter corresponds to

$$\Phi (\hat{\theta}(\tau), s) - C'(\tau) = 0. \quad (4)$$

The lack of concavity of this function implies that in general there might be several values of $\tau$ that satisfy this condition. We denote the one that maximizes utility $\tau$. Notice that this solution is the same for all constrained agents regardless of their level of warm-glow giving $\theta$.

When the constraint is not binding for the voter, the first order condition corresponds to

$$\Phi (\hat{\theta}(\tau), s) - \theta = 0$$

which, given that $\hat{\theta}(\tau)$ is increasing in $\tau$, describes a minimum (the second order condition is positive). Therefore, the optimum must correspond to either $\tau = 0$ or to $\tau = 1$. The second case can be ruled out because for the maximum $\tau$ the constraint would be binding by definition. This analysis leads to the following characterization of $\tau^*(\theta)$.

**Proposition 4**  *The most preferred tax for an agent of type $\theta$ corresponds to*

$$\tau^*(\theta) = \begin{cases} \tau & \text{if } \theta \leq \hat{\theta}(\tau), \\ 0 & \text{otherwise,} \end{cases}$$
where \( \tau \) solves
\[
\tau \in \arg \max_{\tau} \Phi (\theta(\tau), s) \tau + \int_{\theta(\tau)} \hat{g}(x) \phi(x, s) \, dx - C(\tau).
\]

As previously stated, the monotonicity of \( \tau^* \) implies that the equilibrium tax rate of a society corresponds to the most preferred choice of the median voter, \( \tilde{\tau}^*(s) \equiv \tau^*(\tilde{\theta}) \).

We can now illustrate the workings of the aggregate and the median-voter effect. The median-voter effect corresponds to the decrease of \( \tau \) as \( \theta \) increases. With respect to the aggregate effect, an increase in \( s \) results, according to (4), in a lower \( \tilde{\tau} \) and as a consequence a lower \( \tilde{\theta} \), making both lower the tax that a non-altruistic median voter would choose and more likely that the median voter chooses \( \tau^* = 0 \). Hence the equilibrium \( \tilde{\tau}^*(s) \) is decreasing in \( s \).

The previous characterization of the voting equilibrium allows us to explore the effect of increasing altruism (a higher \( s \)) in the provision of the public good. Then, proposition 5 states the main result of the paper: the supply of this public good is not necessarily monotonic with the level of altruism in the society.

**Proposition 5** The equilibrium provision of the public good \( G \) is not in general globally increasing in \( s \).

We prove the previous result by means of example. First, however, in order to provide some intuition, it is useful to decompose the effect of a change in the distribution of \( s \) into two different channels. With some abuse of notation, a change in \( s \) could be described as the following\(^6\)
\[
\frac{dG}{ds} = \frac{\partial G}{\partial s} + \frac{\partial G}{\partial \tilde{\tau}^*} \frac{d\tilde{\tau}^*}{ds}.
\]

From Remarks 1 and 2 we know that \( G \) is increasing in \( s \) for a given \( \tau \), and that it is also increasing in \( \tau \) for a given \( s \). However, we have just shown that the equilibrium tax is lower in more altruistic societies. Therefore, the total effect is undetermined. There exists, nonetheless, an important case in which it is easy to show that \( G \) decreases in \( s \). Suppose that no agent is altruistic (that is, the distribution \( \Phi \) is degenerate at 0), then the equilibrium

\(^6\)Notice that \( G \) is not necessarily differentiable with respect to \( s \). In that case, the same decomposition holds where derivatives are replaced by increments.
tax rate is \( \tilde{\tau}^* = 1 \), which yields the first best level of the public good, since all agents are forced to contribute \( g = 1 \). This is not necessarily true when most agents have \( \theta = 0 \) yet a few agents have a positive value of \( \theta \). In this case the direct effect of \( s \) over \( G \) will be zero, since changes in the distribution cannot lead to an increase in \( G \) beyond its first best level. However, the second term will be negative. The median voter will still be located at \( \theta = 0 \) but he will choose \( \tilde{\tau}^* < 1 \) due to the aggregate effect. In other words, a lower tax level allows him to save on the cost of complying with the tax rate, measured by \( C(\tau) \), which compensates for the lower level of the public good.

Of course, the negative relationship between altruism and the provision of the public good is not general either. Trivially, when the distribution \( \Phi \) is degenerate around 1 even though the tax enacted is 0, the stock of the public good will also be 1.

We end this section by pointing out in the following remark an important part of the previous analysis.

**Remark 3** Increases in altruism lead to increases in voluntary contributions that vote-out mandatory contributions through lower taxes.

We can better explain this result by noticing that the total contribution to the public good can be decomposed as the sum of voluntary and mandatory contributions. That is,

\[
G = \tilde{\tau}^*(s) + \int_{\theta(\tilde{\tau}^*(s))}^{\tilde{\tau}^*(s)} (\tilde{g}(\theta) - \tilde{\tau}^*(s)) \phi(\theta, s) ds.
\]

Suppose that in our model we take \( \tau \) as an instrument of an *exogenous* policy. If we conduct the comparative statics of (exogenously) increasing the tax rate, we obtain an imperfect crowding-out. For those agents that contribute voluntarily, increasing \( \tau \) reduces the voluntary donation by exactly the same amount, and has a neutral effect. In the aggregate, the crowding-out is imperfect due to those agents that are constrained to contribute the mandatory level. This would be an equivalent to the standard result in the literature on philanthropy in which public funding crowds-out private funding.

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7 The reason is that the median voter is the representative agent in the economy and his problem when \( \theta = 0 \) coincides with the planner’s problem. Hence, \( \tau = \arg \max \tau - C(\tau) \).

8 Rather than willing to emphasize the lack of efficiency in the provision of the public good, here we only want to point out the non-monotonicity of \( G \) with respect to the level of altruism. Efficiency in this case is an artifact of the way the first best is defined when preferences display warm-glow giving and the assumption that \( \theta \) cannot exceed one. For an upper bound of \( \theta \) greater (lower) than 1, excessive (insufficient) provision would arise.
Our contribution in this paper has been to make the mandatory contribution (taxes) endogenous to the distribution of altruism (warm-glow giving) in society. We have shown, then, that accounting for this effect yields a voting out of mandatory contributions by voluntary contributions, and that this voting out may be more than one-to-one with a resulting smaller level of the public good.

3 Alternative Preference Specifications

The reduced-form model we have presented in the main section of the paper assumes that agents derive utility only from the private and voluntary part of their contributions (warm-glow giving preferences). In this section, we show that the result that the stock of the public good is not always monotonic (i.e. may decrease) in the level of ”altruism” in the society still holds for other type of preference specifications in which agents have a willingness to contribute to the public good beyond their direct self-interest.

We consider two alternative preference specifications. In the first one, in addition to the utility derived from their voluntary contributions to the public good, agents also derive some utility from the taxes paid for the public good. In the second one, agents’ motive for contribution to the public good lies in the desire to build some reputational signal (à la Benabou and Tirole, 2006). In both types of preference specifications, we show that endogenizing mandatory contributions may generate the same voting out effect analysed above.

3.1 Warm-glow from Giving and from Taxes

In addition to the utility derived from their voluntary contributions (warm-glow giving), in some contexts it might be reasonable to assume that agents may also derive some utility from their mandatory taxes paid to fund the public good (culture, education, resources to the poor, etc.). This is the assumption we discuss in this subsection. More specifically, we consider an agent with the following utility function

\[ U(g, \tau) = G + \theta (g - \beta \tau) - C(g), \]

where the parameter \( \beta \in [0,1] \) measures the difference in weights between the voluntary and mandatory contributions. To better interpret this parameter we can decompose the effect of
\( \theta \) as follows,

\[
\theta(g - \beta \tau) = \theta [(g - \tau) + (1 - \beta)\tau],
\]

where the first term in brackets corresponds to the voluntary contribution of the agent and the second is the mandatory contribution. Preferences that exhibit warm-glow voluntary giving (as in the preceding section) correspond to the case of \( \beta = 1 \). The limit case \( \beta = 0 \) is consistent with preferences under which the warm-glow giving utility of the agent is independent of whether the contribution is voluntary or mandatory.

Under this more general specification, the second stage of the game remains unchanged for a given \( \tau \). It is still true that increases in \( \tau \), or a more altruistic society, lead to a higher supply of the public good. In the voting stage, it can still be shown that utility is still submodular in \( s \) and \( \tau \), and the aggregate effect persists for any \( \beta \). However, utility fails to be submodular in \( \theta \) and \( \tau \) and it does not even satisfy a weaker single-crossing condition.

**Lemma 6** For \( \beta \in (0, 1) \), the indirect utility function

\[
V(\theta, \tau, s) = \int g^*(x) \phi(x, s) \, dx + \theta (g^*(\theta) - \beta \tau) - C(g^*(\theta))
\]

is submodular in \( s \) and \( t \) but it generically fails to be single-crossing in \( \theta \) and \( \tau \).

An implication of this result is that we cannot apply the Median Voter Theorem. Let’s see the intuition. Consider two agents with different intensity in their warm-glow giving parameter \( \theta \). First, suppose that both agents’ preferred choice is to set a \( \tau \) such that both agents are constrained (they will only give the mandatory contribution). Recall that the warm-glow giving can be decomposed as \( \theta [(g - \tau) + (1 - \beta)\tau] \). Then in the present case, the agent’s marginal utility of increasing \( \tau \) due to warm-glow is \( \theta(1 - \beta) \) since the voluntary part is zero. Thus, given that the effect of the total good is identical across the population, it is clear that an agent with a larger \( \theta \) prefers a larger \( \tau \).

Second, suppose that both agents’ preferred choice is to set a \( \tau \) such that neither agent is constrained (they will both provide positive voluntary contributions). Then, in the present case we can see that changes in \( \tau \) do not affect their total contribution. However, changes in \( \tau \) affect the composition of funding to the public good between voluntary and mandatory contributions. For \( \beta > 0 \), the warm-glow giving utility is larger for the voluntary part than the mandatory one. As a consequence, an agent with a higher \( \theta \) suffers more from a higher \( \tau \) and would thus prefer it to be lower.
For $\beta \in (0, 1)$, there will always be constrained and unconstrained agents, and the Median Voter Theorem cannot be generically applied. However, with $\beta = 1$, which was the focus of this paper in the preceding section, monotonicity holds since agents only obtain warm-glow utility from voluntary contributions. Hence, the first case scenario above does not arise (the preferred $\tau$ cannot be strictly increasing in $\theta$).

Analogously, with $\beta = 0$, agents value equally mandatory and voluntary contributions and the second case above does not arise (the preferred $\tau$ cannot be strictly decreasing in $\theta$). As a matter of fact, in this case it can be shown that $V$ is supermodular in $\theta$ and $\tau$. Thus, the preferred tax level is increasing in $\theta$, and this monotonicity allows us to apply the median-voter theorem.\(^9\) Notice that with $\beta = 0$ the median-voter effect operates in the opposite direction. At the same time, if society becomes more altruistic, the aggregate effect leads to a lower tax rate. The sum of the two countervailing effects implies that the total effect of a shift in the distribution over the equilibrium tax rate remains in general undetermined. Still we can show that a counterpart of Proposition 5 holds in this case. For example, when all agents are not altruistic, the stock of the public good will still be optimal. A shift in the distribution will be dominated by the aggregate effect to the extent that it does not affect the median voter. In this case, the equilibrium level of the public good will decrease due to the decrease in $\tau$.

Finally, for intermediate values of $\beta$ even though we cannot provide a general result due to this lack of regularity of the voting game, we can show that Proposition 5 still applies at least in a particular setting. When the distribution of $\theta$ is binomial, the median voter is trivially defined.

**Example 1 (The Binomial Case)** Assume $\Phi$ is a two-point distribution so that $\theta = 1$ with probability $\alpha$, while $\theta = 0$ with probability $1 - \alpha$. In the second stage of the model, for a given $\tau$, $\hat{g}(0) = 0$ and $\hat{g}(1) = 1$. Therefore, $g^*(0) = \tau$ and $g^*(1) = \max\{1, \tau\}$. The equilibrium level of $\tau$ (determined at the voting stage) depends on the proportion of voters of each type.

If $\alpha < \frac{1}{2}$ the median voter corresponds to $\tilde{\theta} = 0$. In this case, the problem that the median voter solves is

$$\max_{\tau} (1 - \alpha)\tau + \alpha g^*(1) - C(\tau).$$

\(^9\)This environment is analyzed in a different context by Calveras et al. (2006) and we refer the reader to this paper for a more in-depth description of the model.
The first order condition of this problem defines the equilibrium tax level as

\[ 1 - \alpha = C'(\tau^*), \]

and, therefore, a lower than optimal tax is enacted (given that \( C'(\cdot) \) is convex, and \( C'(1) = 1 \)). Since the median voter remains unchanged for all \( \alpha \) in this range, the aggregate effect dominates and the equilibrium tax rate decreases in more altruist societies.

If \( \alpha > \frac{1}{2} \) and the median voter has \( \tilde{\theta} = 1 \), his maximization problem corresponds to the following expression

\[ \max_{\tau} (1 - \alpha)\tau + \alpha g^*(1) + (g^*(1) - \beta \tau) - C(g^*(1)). \]

Two possibilities need to be considered for the first order condition depending on whether \( \tau \) is less or greater than 1. In the second case, the first order condition leads to an optimal value of \( \tau \) solving

\[ (2 - \beta) - C'(\tau^*) = 0, \]

with \( \tau^* \) greater than 1. In the first case, the first order condition

\[ 1 - \alpha - \beta \]

is linear, leading to a corner solution. When \( \beta < 1 - \alpha \), \( \tau = 1 \) is optimal in this range. However, this is a particular case of the previous problem when \( \tau \geq 1 \) which has an interior solution, and this solution prevails.

When \( \beta > 1 - \alpha \), the opposite corner, \( \tau = 0 \) competes with the interior solution for \( \tau \geq 1 \) to be the global optimum. It can be shown that there exists a critical value \( \bar{\alpha}(\beta) \) decreasing in \( \beta \) such that for \( \alpha > \bar{\alpha}(\beta) \), \( \tau = 0 \) dominates; while for \( \alpha < \bar{\alpha}(\beta) \), \( \tau^* > 1 \) dominates. Figure 1 illustrates the three possible configurations of the equilibrium.

Notice that, in spite of the complexity of the equilibrium tax rate when the median voter is altruistic, one result is always robust in this example independently of \( \beta \). When the median voter is not altruistic an increase in altruism reduces the tax rate and also the stock of the public good.

Overall, then, we shown that when mandatory contributions are endogeneized in the case in which agents have warm-glow both from voluntary giving and from taxes, a more altruistic society may also end-up with a lower level of provision of the public good.
Figure 1: The stock of the public good for different specifications of the cost function. The first case is consistent with a low value of $\beta$ and the third with $\beta$ close to 1. The intermediate case is displayed in the other figure.
3.2 Reputational Giving

Alternatively to the rest of the paper, suppose that agents do not derive utility from warm-glow giving per se (neither from their voluntary nor from their mandatory contribution). Instead, the utility derived from their contribution depends on how agents regard themselves with respect to the society as a whole; that is, they may derive different utility from the same amount of contribution depending on the contributions made by others. This prosocial behavior has been studied in the literature by Benabou and Tirole (2006) in a full-fledged model. Here, we use a simple interpretation of such a framework and we posit agents’ preferences according to the following utility function

\[ U(g, G) = G + \theta(g - \bar{g}) - C(g), \]

where \( \bar{g} \) is the average contribution in society to the public good. An immediate interpretation of these preferences is to consider that reputation is built by signaling contributions beyond the average contribution in the society, \( \bar{g} \).\(^{10}\) We assume that the parameter \( \theta \) is distributed according to \( \Phi(\theta, s) \) and we bound it, so that it is always non-negative and no agent would individually make voluntary contributions beyond the efficient level. In other words, \( \theta \in [0, 1] \).

Due to the unit mass of agents in the economy, the average contribution also coincides with the total contribution, \( \bar{g} = G \). We restrict contributions to be above a minimum \( \tau \), the prevailing tax level. As in the rest of the paper, the model proceeds in two stages. Voters elect a tax rate and they later choose their voluntary contributions.

Given the negligible impact that individual contributions have on the total stock of the public good (and, thus, on the average contribution in society), it is immediate that the decision in the second stage will be defined by \( g^*(\theta) \) as it is specified in (1). In that expression \( \hat{g}(\theta) \) corresponds to the unconstrained voluntary contribution. Similarly, there exists a critical value of \( \theta \) such that the tax level \( \tau \) is binding only for lower degrees of the prosocial behavior. As earlier, this threshold value is denoted as \( \theta_\tau \) and it is still defined by (3). The level of the public good in the first stage given the optimal decision of all agents in the second stage, \( \hat{g} \), can again be written as

\[ G = \Phi(\theta(\tau), s) \tau + \int_{\hat{g}(\tau)} \hat{g}(x) \phi(x, s) \, dx. \]

\(^{10}\)This interpretation does not preclude alternative rationalizations of these preferences. For example, even if voluntary contributions are only driven by self-motivation, the framing provided by other agents’ actions is useful in assessing the goodness of individual actions. In this last interpretation the parameter \( \theta \) would be closer to the warm-glow giving studied earlier.
Replacing \( \hat{g} \) in the utility function, the following expression transpires,

\[
V(\theta, \tau, G) = G + \theta(g - \Phi(\theta(\tau), s) \tau) - \theta \int_{g(\tau)} \hat{g}(x) \phi(x, s) \, dx - C(g).
\]

We can now turn to the decision regarding the tax rate in the first (voting) stage. Contrary to the previous analysis in which agents enjoyed warm-glow from giving, we can now show that a voter’s most preferred tax, if implemented, would lead him to no voluntary contribution. That is, a voter is constrained by his preferred tax level \( \tau^*(\theta) \). In order to show this result, suppose otherwise, and in particular, assume that \( g^*(\theta) > \tau^*(\theta) \). Differentiating the voter’s utility function, we obtain that

\[
\Phi(\theta(\tau), s)(1 - \theta) > 0,
\]

which contradicts the previous assumption. The intuition for this result lies in that an increase in the tax rate decreases the utility from the reputational giving at a lower rate than the increase in the utility derived from the public good. This result allows us to state the following properties of the function \( V \).

**Lemma 7** The function \( V \) is

1. submodular in \( s \) and \( \tau \), and
2. supermodular in \( \theta \) and \( \tau \).

Notice that the first property corresponds to the standard aggregate effect, while the second one means that the median-voter theorem applies: the implemented tax will thus be the most preferred one of this median-voter. Notice also that the median voter effect in this setup goes in the opposite direction as in the benchmark model. That is, more prosocial voters will choose a higher tax rate. The reason is that, using the previous arguments, the median voter will always make a total contribution below the mean, \( \bar{g} \). However, a higher \( \tau \) reduces this prosocial disutility, which is more valued by agents with a larger \( \theta \).

Because the two effects go in opposite directions, the total effect over the tax rate of moving to a more prosocial society will be in general ambiguous. However, we see next that a crowding out of \( G \) by voluntary contributions can also occur in this case with these prosocial preference specification.
Example 2  Suppose that \( \theta \) has a two-point support. A proportion \( 1 - \alpha \) has \( \theta = 0 \) while a proportion \( 1 - \alpha \) has \( \theta = \bar{\theta} > 0 \). The most preferred \( \tau \) for an agent of type \( \theta = 0 \) solves

\[
\max_{\tau} \alpha g^*(\bar{\theta}) + (1 - \alpha)\tau - C(\tau)
\]

with first order condition

\[
(1 - \alpha) - C'(\tau) = 0
\]

that implies \( \tau^* \leq 1 \) and decreasing in \( \alpha \). From the previous arguments agents with \( \theta = \bar{\theta} \) choose \( \tau > g(\bar{\theta}) \), and his prosocial utility becomes 0. His problem becomes the social planner problem given by

\[
\max_{\tau} \tau - C(\tau)
\]

As a result, the equilibrium tax rate in this problem can be represented as the first part of Figure 1 where as opposed to that case, the tax never exceeds 1.

4 Concluding Remarks

Many papers have emphasized the fact that the individual voluntary contribution to public goods depends on the level of taxation prevalent in the society. In this paper we have shown that there is also feedback in the other direction: an agent’s voting decision depends on his/her desire to contribute to the public good (altruism) as well as the philanthropic desire of the other agents in the society. Moreover, this feedback may lead to counterintuitive results. Namely, the equilibrium level of the public good may be non-monotonic in the level of altruism. This puzzling result is due to the interaction of two effects.

First, there is an aggregate effect. Agents contribute to the public good both through voluntary donations and taxes. Due to the known crowding-out of private contributions when taxes increase, the more altruistic is the agent the less sensitive is his total contribution to taxation.\(^{11}\) Consequently, in a more altruistic society the effectiveness of taxes as a tool to increase the level of the public good is reduced.

Second, there is a median-voter effect. Obviously, in a more philanthropic society, the median voter, whenever it is well defined, will be a more altruistic agent. The way this more

\(^{11}\)We have modeled the different elasticity of the total contribution of agents to changes in altruism in a very stark way. Some agents make no voluntary contributions and therefore they are constrained by the mandatory tax level, whereas some others are not constrained and their voluntary contribution is reduced one-by-one with increases in the tax rate. Hence, in a more altruistic society fewer agents are constrained and the productivity of increasing the tax rate is lower.
altruistic preference translates into his voting decision depends in the source of his altruistic behavior. This paper provides examples in both directions. When agents are motivated by warm-glow giving preferences, and mainly obtain utility from their voluntary contributions, a more altruistic voter is likely to choose a lower tax rate. However, when agents are motivated by their total contribution or by their contribution relative to others, that we denote as reputational concerns, the median-voter effect goes in the opposite direction.

A lesson from this paper is that the source of the non-monotonicity is the opportunistic behavior that the aggregate effect promotes, regardless of the direction of the median-voter effect. In a more altruistic society, a lower tax level, and therefore a lower public funding of the public good, is likely to arise. Moreover, we have shown that the aggregate effect might be sufficiently strong to lead to an increase in the level of the voluntary contribution that cannot make up for the decrease in the resulting public funding. Hence, the total level of the public good will decrease when this more than one-to-one reverse crowding out effect occurs.

The model we have presented is particularly convenient in order to isolate the interaction between private and public contributions. However, it abstracts from many relevant dimensions. We do not consider explicitly the differences in income or preference for the public good across agents. In this respect our measure of altruism could be a composition of these as well as other dimensions of heterogeneity. Moreover, this public good might be subject to decreasing returns to scale and might be financed with distortionary and probably progressive taxes.

Further research in a more comprehensive model of voting behavior is required. However, we can use our decomposition of the consequences of altruism between the median voter and aggregate effect to assess the likely implications of an increase of altruism in the society. As we have already shown, the median voter effect is very sensitive to modeling decisions and its implications are difficult to predict. To the contrary, the aggregate effect is based on the fact that a larger volume of the voluntary contributions reduces the power of taxes to increase the provision of the public good. This effect seems robust to alternative modeling specifications.
A Proofs

**Proof of Lemma 1**: Consider the following difference in utility for \( s' > s \),
\[
V(\theta, \tau, s') - V(\theta, \tau, s) = \Phi(\theta(\tau), s') \tau + \int_{g(\tau)} \hat{g}(x) \phi(x, s') \, dx - \Phi(\theta(\tau), s) \tau - \int_{g(\tau)} \hat{g}(x) \phi(x, s) \, dx.
\]
Deriving the expression with respect to \( \tau \) we obtain
\[
\frac{d}{d\tau} (V(\theta, \tau, s') - V(\theta, \tau, s)) = \Phi(\theta(\tau), s') - \Phi(\theta(\tau), s) < 0
\]

**Proof of Lemma 2**: Consider the following difference in utility,
\[
V(\theta', \tau) - V(\theta, \tau) = \theta' (g^*(\theta') - \tau) - C(g^*(\theta')) - (\theta (g^*(\theta) - \tau) - C(g^*(\theta))).
\]
We need to show that this expression is decreasing in \( \tau \). Three cases need to be considered:

1. \( \hat{g}(\theta') < \tau \) so that the constraint is binding for both types. Then \( V(\theta', \tau) - V(\theta, \tau) = 0 \).
2. \( \hat{g}(\theta) < \tau < \hat{g}(\theta') \) so that the constraint is only binding for the low type. In that case
\[
V(\theta', \tau) - V(\theta, \tau) = \theta' \left( \hat{g}(\theta') - \tau \right) - C(\hat{g}(\theta')) + C(\tau)
\]
and this expression is decreasing in \( \tau \) since using the concavity of the second stage utility function, it has to be that for all \( g < \hat{g}(\theta') \)
\[
\theta' g - C(g)
\]
is increasing in \( g \). Therefore, \(-\theta' \tau + C(\tau)\) is decreasing in \( \tau \).

3. \( \tau < \hat{g}(\theta) \) where \( \tau \) is not binding for any of the firms. Hence, the function is decreasing given that the slope with respect to \( \tau \) is \( \theta - \theta' < 0 \).

**Proof of Proposition 4**: In the text.

**Proof of Lemma 6**: For the first part see proof of Lemma 1. The second part is immediate from the arguments in the tex.

**Proof of Lemma 7**: The optimal \( \tau \) for type-\( \theta \) voter provided that the tax is binding for him solves
\[
\tau^* \in \arg \max_{\tau} G + \theta \left( 1 - \Phi(\theta(\tau), s) \right) \tau - \theta \int_{\hat{g}(\tau)} \hat{g}(x) \phi(x, s) \, ds - C'(\tau).
\]
with first order condition
\[
\Phi(\theta(\tau), s) + \theta \left( 1 - \Phi(\theta(\tau), s) \right) - C'(\tau) = 0
\]
Notice that this derivative is decreasing in \( s \) and increasing in \( \theta \) which is enough to prove the result.
References


