Optimal income taxation of lone mothers: an empirical comparison for Britain and Germany

by

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Abstract:

In this analysis we apply the optimal tax rule suggested by Saez (2002) to empirically discuss the optimal tax and transfer design for lone mothers in Germany and in Britain. The key advance of this paper is that we combine the theoretical model with a structural estimation of households` labour supply. Thus we are able to allow for heterogeneity between groups regarding their behaviour adjustment rather than calibrating an overall labour supply elasticity for the whole society. We find that in-work credits for lone mothers are optimal from a social welfare perspective with relatively low and medium taste for redistribution in both Germany and Britain. Further, we show that the current design of the tax and benefit system in both countries, without an explicit in-work credit, is only optimal if the government has a high welfare value for the non working lone mothers and a relatively low taste for redistribution towards the working lone mothers. Our results are driven by relatively high elasticities on the extensive margin which implies a high positive participation response of the non working.

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1 Introduction

Government manages transfer and redistribution programmes are of major importance in most developed countries. Almost all these countries spend large amounts of public funds to provide income support to the poor, although the structure of these programmes differs substantially across countries. As expenditures on public income support programmes count for a sizeable share of the government's budget in welfare states, and because of their alleged negative work incentive effects, there is an ongoing public debate about policy reforms in this area. This controversy can be best described by the trade off between *equity and efficiency*. Whereas income transfers increase the disposable income of the disadvantaged, and thus increases their well-being, these programs introduce distortions that might lead to substantial disincentives on the labour market.

Individuals can adjust their labour supply along two margins, i) the decision to participate on the labour market (extensive margin), and ii) the decision about working time (intensive margin). Although labour supply effects on the extensive margin tend to be more important (Heckman, 1993) it is necessary to study the intensive margin as well when analysing the labour supply behaviour. This is in particular important for the evaluation of welfare programs such as “making work pay” policies as these reforms might provide opposite incentives for the labour market participation and the working hours.

The design of transfer programs, and the trade-off between equity and efficiency has been intensively analysed in the economic literature. The seminal theoretical contribution is Mirrlees (1971). In that framework, which focuses exclusively on the case where agents chose how much to work (i.e., on the intensive margin), it can be shown that negative marginal tax rates can never be optimal, ruling out in-work credits.[1] Diamond (1981) extended the model of optimal income taxation by focussing only on the extensive labour supply margin. In this framework, the optimality results derived within the Mirrlees framework no longer hold. Instead, Diamond shows that for some income ranges, optimal marginal taxes may be negative. Saez (2002) suggested a model that combines the ideas in both Mirrlees and Diamond, and allows for workers to choose whether and (to a degree) how much to work; he

[1] In this paper, we use the phrase “in-work credit” to mean a tax system that redistributes more to people with strictly positive earnings than it does to those who do not work. Thus this tax system includes negative marginal tax rates.

shows that it is more likely that optimal tax rates may turn negative the larger is the extensive elasticity relative to the intensive elasticity.

The aim of this paper is to apply the theoretical model presented in Saez (2002) to analyse empirically the design of income taxation, and to discuss its optimality. We focus on the tax and transfer systems in Germany and the UK. More specifically, we want to assess and compare the design of the tax and transfer system for lone mothers in both countries.

We chose to focus on lone mothers for a number of reasons. First, in both countries, lone mothers are eligible for generous transfer programmes, and the interaction of transfer programmes and the income tax system can generate budget constraints with high and variable effective marginal tax rates. Second, there is a (partly emotional) debate in both countries about the extent to which lone mothers should be supported by the state, even when they do not work. This is in particular true for lone parents with pre-school age children. Moreover, in practical terms, focusing on lone adult households allows us to avoid the substantial complexity to both models of labour supply as well as optimal tax theory that arise when dealing with household decisions of labour supply. So far the optimal tax literature has not suggested a theoretical framework accounting for the simultaneous decision of households that can be empirically analysed.[2] Lastly, concentrating on lone mothers who are in general a relatively low-skilled, low-wage group gives greater justification to studying exclusively labour supply responses to taxation, rather than responses involving other factors that might affect taxable earnings (Gruber and Saez, 2002).

In both, Britain and Germany, lone mothers are of important size. According to the German population survey, in 2003 more than 16% of all families with the youngest child below 18 years are households with a single parent (Statistisches Bundesamt, 2004). This implies that about 15% of all children younger 16 are raised by single parents. In Britain the share of lone parents is even higher: Roughly 25% of families with children are lone parents (Office for National Statistics 2005). In both countries the majority of lone parents are mothers, a minor share of less than 10% of lone parents household have only a father. Therefore, we focus solely on lone mothers.

[2] In a recent study Kleven et al. (2006) suggest a theoretical framework for the optimal taxation of couple households.

We analyse the optimality of income taxation in a comparative setting for Britain[3] and Germany. A comparison between these countries is interesting for several reasons. Most important, the transfer and benefit systems for lone parents are quite different in both countries. In Britain, there is a clear dichotomy between out-of-work support provided through traditional means-tested benefits and in-work support, provided through refundable tax credits, and the importance of the latter has grown substantially over the past decade. Germany, in contrast, relies on the more traditional means-tested social assistance which is designed with an extra supplement for lone parents, causing very high positive marginal withdrawal rates at the bottom of the earnings distribution. Yet, as we show later, the budget constraints facing lone mothers in the two countries have in practice are similar design, even if different ideas underpin the two transfer systems. Most importantly real in-work credits, implying higher transfers for the working than for the non-working (i.e. negative tax rates) are not part of the tax and transfer system neither in Britain nor in Germany. Between both countries though, there exist enormous differences in the labour market behaviour of lone parents. Whereas the overall female labour market participation of women tends to be higher in Britain, British lone mother have relative low participation rates in comparison to lone mothers in Germany.

In this study, we address two questions. First, following Bourgignon and Spadaro (2005), we want to assess the welfare weights that a social planner would assign to different groups given that the tax and transfer system that we observe in each country is optimal, and given the labour supply elasticities that we estimate. Second, we want to derive the optimal tax schedules in each country given various assumed social welfare functions. As we find strong differences in the employment of lone mothers with and without pre-school children, we provide a separate analysis of optimal taxation by the age of the youngest child.

Based on the theoretical literature of optimal taxation, there exist several empirical studies that analyse and compare welfare and tax systems of different countries.[4] Immervoll et al. (2006) apply a basic framework of optimal taxation to the analysis of two different transfer programmes for 14 Western European countries: the first reform is traditional means tested welfare that covers all; the second reform proposal is an in-work tax credit that focuses

[3] In this study we use Great Britain (ie without Northern Ireland) and the United Kingdom exchangeable. The employed data is from Great Britain but the policy (taxes, tax credits and benefits) applies across the UK.

[4] There exists numerous empirical studies on welfare effects of tax reforms (e.g. Aarberge and Columbino, 2005). However these studies differ from the models closely linked to the optimal income tax theory as they are not derived from an optimal tax formula but rather from structural econometric models of labour supply behaviour.

exclusively on the working poor. The authors use the microsimulation model EUROMOD that mimics the current welfare and tax system of 14 European countries, and calibrate labour supply elasticities on the intensive and extensive margin. Their results are strongly in favour of the in-work tax credit: they conclude that in particular in countries with large welfare programmes, such as Germany, a purely means-tested benefit programme is not desirable. Eissa, Kleven, Kreiner, (2005) evaluate the welfare effects of four tax reform acts on single mothers in the United States over the last 20 years. They find that the tax reforms reduced the tax burden for this group and thereby causing welfare gains. Yet, as in Immervoll et al. (2006), this study does not allow for heterogeneity in the behaviour of individuals but assumes labour supply effects to be constant at some given rate.

Thus, the key advance in this paper, in contrast to the previous literature on optimality of the tax and benefit system, is that we combine the theory of optimal taxation with both country-specific tax and benefit microsimulation models and country-specific structural models of labour supply. This enables us to recognise fully the complexity and heterogeneity in the tax and transfer system within each country, and it also allows us to estimate, rather than calibrate, the key behavioural inputs in the expression for optimal tax rates, namely labour supply elasticities. Hence, the extension of the employed method implies that the heterogeneity in household behaviour is accounted for which is according to Saez (2002) crucial for the analysis of the optimal tax design.

2 The theoretical model

We base our analysis on the framework outlined in Saez (2002), slightly modified for our research questions. Generally, the problem of optimal income taxation can be described as follows: a social planner (the government) maximises a social welfare function given its budget constraint. The social welfare function is a transformed function of individual utilities which themselves depend on net household income (consumption) and leisure. The functional form of the social welfare function is based on normative assumptions ranging from a Rawlsian to a Utilitarian welfare function. In a Rawlsian society, the social planner cares only about the worst off individual; in an Utilitarian world, the social planner weights the utility of all individuals equally.

In the framework of optimal taxation, the margin along which individuals can adjust their behaviour is their labour supply. This leads to the trade-off between equity and efficiency. Whereas transfer programs can increase the disposable income of the disadvantaged, and thus increase their wellbeing, financing these programs with positive income tax rates introduces disincentives to work, and, in general, will lead to a reduction in labour supply of the working population.

Saez (2002) sets up an optimal tax problem where there are *I+1* discrete groups in the labour market: *I* groups of individuals who do work, plus one group consisting of those who do not work. Individuals choose whether or not to participate (the extensive margin), and which group to choose (the intensive margin). In this framework, optimal taxation has the following form:

$$\frac{T_i - T_{i-1}}{C_i - C_{i-1}} = \frac{1}{\mu_i h_i} \sum_{j \geq i}^{I} h_j \left[1 - g_j - \eta_j \frac{T_j - T_0}{C_j - C_0} \right].$$

In this expression, T_i is net tax paid by group i and C_i is the net household income of this group, so the term on the left-hand side is the extra tax paid when moving from group i-1 to i divided by the gain in net income. Non-workers receive benefits $-T_0$, by definition identical to C_0. The gross earnings of group i, equal to $C_i + T_i$, are exogenously fixed. h_i measures the share of group i in the population. The social welfare function is summarised by g_i, the weight the government assigns to group i.

The intensive elasticity or mobility elasticity (Saez, 2002), μ_i, is defined as:

$$\mu_i = \frac{C_i - C_{i-1}}{h_i} \frac{dh_i}{d(C_i - C_{i-1})}.$$

This mobility elasticity captures the percentage increase in supply of group i when C_i-C_{i-1} is increased by 1%. It is defined under the assumption that individuals are restricted to adjust their labour supply only to the neighbouring choice.

Finally, η_i is a measure of the extensive elasticity, and is defined as the percentage of individuals in group i who stop working when the difference between the net household income out of work and at earnings point i is reduced by 1%:

$$\eta_i = \frac{C_i - C_0}{h_i} \frac{dh_i}{d(C_i - C_0)}.$$ [5]

The main implication of the optimal tax rule above is that the optimal tax system depends heavily on whether labour supply responses are concentrated at the intensive or extensive margin. When the extensive elasticity is assumed to be zero, Saez' model gives results similar to Mirrlees', where negative marginal tax rates are never optimal. However, the greater is the extensive elasticity compared to the intensive elasticity, the more likely it is that the optimal schedule will feature relative smaller guaranteed income for non-workers, and negative marginal taxes at low levels of earnings.

We apply the model outlined above to a comparative analysis of optimal income taxation. The focus of this analysis is on the tax and transfer system of lone mothers in Germany and the UK.[6] As Saez (2002), we define the groups by gross earnings. A first-best solution of income tax would be based on measures of skill or productivity captured by the hourly wage, but in practice this cannot be observed, and so optimal tax models assume that the income tax has to be a function solely of gross earnings. We condition the optimal income schedule on this information as we aim to mimic the taxation decision the government faces.

3 Lone mothers in Germany and UK: The tax and transfer system and labour market behaviour

Overall, the female employment rate[7] is with 66.3% higher in the UK than in Germany, where 58.8% of the relevant population is employed (OECD, 2005). However, as Haan and Myck (2006) show, the picture is different for lone mothers. This is partly due to compositional differences - lone mothers in Germany have older children – but it is also due to other factors: conditional on the age of their children, lone mothers in Germany work more than in the UK.

[5] As we show empirically in the following section, this is different from the conventional extensive elasticity, or elasticity of labour force participation, which is (usually) defined as the proportional increase in workers when net incomes rise by 1%.

[6] At first glance it might seem problematic to derive an optimal tax schedule for a sub population. However, the government can distinguish lone mothers and explicitly targets transfers towards this group: income tax legislation in Germany and the UK discriminates between households with and without children, and by marital status. In other words, in this analysis we derive a tax schedule for single adults with children, taking taxation of the rest of the population as exogenous and constant.

[7] Employment rates are defined as the share of employed and self-employed people over the whole population in this age group.

Table 1 gives more detail about the employment behaviour of lone mothers, based on the samples used in the subsequent analysis.[8]

Table 1a: Lone mothers by age of child. Germany

	Share	Employment Rate	Working Hours (Conditional)	Low Education	Age
with children <17		71.37	29.81	34.55	39.2
with children: youngest 0-3	9.06	27.96	19.90	40.29	32.86
with children: youngest 4-6	19.18	53.81	27.05	36.78	33.56
with children: youngest 7-16	71.76	81.55	30.92	33.11	41.72

Note: In Germany, roughly 16% of families with children are lone parents households. Working hours are conditional on employment. Low education is defined as having no degree or only a Hauptschul degree. Source: GSOEP 2002-2004.

Table 1b: Lone mothers by age of child. Britain

	Share	Employment Rate	Working Hours (Conditional)	Low Education	Age
with children <17		52.53	29.02	68.84	35.04
with children: youngest 0-3	27.28	28.82	24.49	68.51	27.85
with children: youngest 4-6	19.35	48.63	25.25	68.96	32.04
with children: youngest 7-16	52.79	65.46	31.16	68.98	39.93

Note: In Britain, roughly 25% of families with children are lone parents households. Low education is defined as having no degree or having left at minimum school leaving age (16 for most of this sample). Source: FRS 2002/3.

In both samples, only a small minority of lone mothers with young children (defined as "any children under 4") work: 29% in the UK and 28% in Germany. But employment rates are markedly higher in Germany than in the UK once children start school: 65% of lone mothers with no children under 7 in the UK work, compared with 81% in Germany.[9]

However, an important compositional difference is that lone mothers in the UK tend to have younger children than in Germany. In our samples, three times as many lone mothers in the UK have children under 4 than in Germany (28% compared with 9%), and almost a half of lone parents in the UK have a child under 7, compared to just under a third in Germany. This considerable compositional difference gives an additional reason why the mean employment

[8] The population for our analysis is lone mothers aged between 17 and 60 with at least one dependent child under 17 years, but excluding self-employed, and those in full-time education. Additionally, the UK sample excludes those receiving disability benefits, and the German sample excludes those declaring themselves to be retired. More information about the data employed and the sample is provided in the Appendix.

[9] Children in the UK start full-time education no later than the term after their 5th birthday; in Germany, school starts in general after the 6th birthday.

rate of lone mothers in the UK is considerably lower than in Germany (52% compared with 71% in our sample).

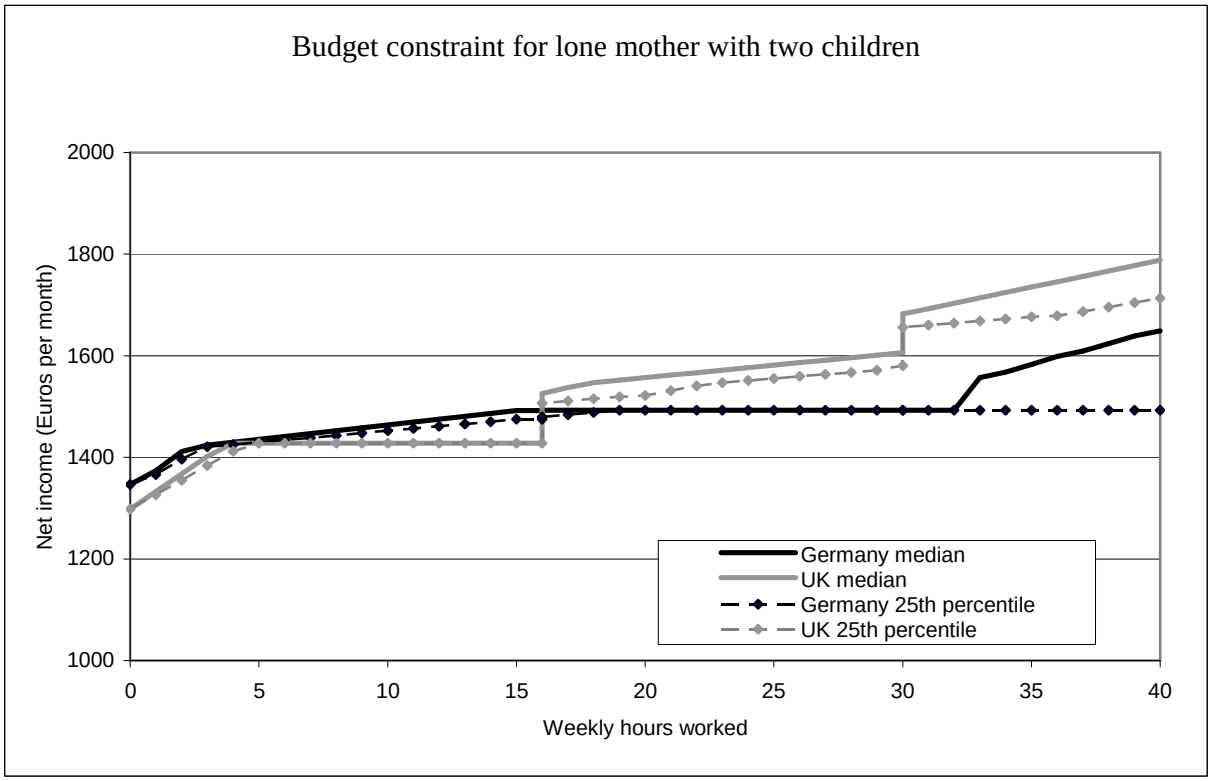
Regarding the working hours, we observe a slightly different picture when comparing the two countries. In general, average working hours for lone mothers conditional on employment are fairly similar in both countries. Decomposed by age of the youngest child we find that working hours are increasing with the age of the child in both countries.

In the last two columns we present the education and age of the single mothers by the age of the child. The difference between the countries is striking. In general we find that lone mother in the UK are younger and have a lower education. In the UK about 2/3 of the lone mothers have a low educational degree and this is about the same regardless of the age of the child. In Germany, in contrast, the overall share of lone mothers with low education is about 1/3 and changes by the age of the youngest child. The older the youngest child the lower is the share of low educated in Germany. The age pattern is as well different in both countries. On average lone mothers are about 4 years older in Germany than in the UK. These differences are important for the comparison of the optimal tax and transfer system between both countries.

A comparative analysis of the tax and benefit design and its optimality in Germany and the UK is insightful as there exist substantial differences in the transfer and benefit systems. In the UK, as well as means tested out of work benefits, a large amount of transfers are made conditional on working through in-work transfers (WFTC during the period covered by our data). In contrast, the German tax and transfer system almost exclusively relies on more traditional means-tested social assistance, with very high withdrawal rates. Thus, the German transfer system is mainly targeted towards the non working poor. This difference is in particular strong for lone mothers, as in both countries several programs are specifically targeted at this group. In the UK, the amount of financial support through in-work credits is dependent on the number of dependent children and as this transfer is withdrawn based on household income, it affects single and couple household differently. The latter is true as well for the means tested income support programs in the UK. In Germany, both income taxation and transfer programs target lone parents differently to couple households with children. There exists an additional tax exemption for taxable income that is conditioned on being lone parent. For single households with children means tested benefits are more generous due to an extra transfer.

The effects of the tax and transfer system on the net household income for lone mothers is best described by looking at stylised budget lines for these families.

Figure 1: Budget constraints for lone mother with two children in UK and Germany, 2002



Notes: For each country we consider a lone mother working at 25th percentile, and at the median hourly wage, renting at the cost of median rent. 25th percentile wage for lone mothers in Britain is €6.45 and in Germany €8.80 median wage is in Britain €7.96. and in Germany €10.90
Source: authors' calculations using TAXBEN and STSM.

Figure 1 presents comparisons of budget constraints for a lone mother with two children for the fiscal year 2002. The budget lines are drawn under the assumption that the woman is earning the 25th percentile or the median female gross hourly wage. At the lowest levels of earnings, i.e. in scenarios where the family qualify for the basic means tested support, disposable incomes of families in Germany and the UK are very similar. Differences become apparent only at hours levels beyond about 16 per week. As mentioned above the transfer system in Germany is mainly based on means tested benefits that are withdrawn with almost 100%. Therefore, the budget line for a low wage lone mother with two children is hardly affected by her working hours. Still at 40 weekly working hours she receives full means tested benefits and her earnings are completely withdrawn. The budget constraint of a comparable lone mother earning median wage is similar. Her net household income starts to increase only after about 32 working hours when the means tested benefits run out. In contrast, the budget line of a low wage mother in the UK is affected by her labour supply. This is the result of

generous in-work support which these families are eligible for in the form of the WFTC. This is true regardless of earning the 25 percentile or the median wage. Interesting to note is that despite wages are markedly higher in Germany, the net household income of lone mothers is higher after working more than 16 hours. This is the effect of the in-work credits. As we will show in the next section, taking all transfer programmes, the UK government is more generous towards lone mothers as the German government is. Over most of the earnings distribution, lone mothers in the UK receive higher transfers than they pay in income and payroll taxes. On average the British government transfer about 200 Euros per week to a lone mother, in Germany the average transfers are with 85 Euros per week markedly lower.

For the empirical analysis we employ detailed country specific microsimulation models, TAXBEN for the UK and STSM for Germany that allow us to derive the amount of tax payments and transfers and the resulting disposable net household income for all lone mothers.[10] This allows us to derive the net income distribution for the lone mothers under the current tax legislation and hypothetical reform scenarios which is necessary to derive the optimal tax schedule for this group. When simulating the net household income we explicitly model child care cost which can be of substantial size and are by definition important when modelling the behaviour of lone mothers. In Germany child care is heavily subsidised, yet availability of child care slot is scarce. Therefore, we follow Wrohlich (2006) and estimate the expected chid care cost according to regional availability of child care facilities. In the UK, we estimate a relationship between hours of work and hours of childcare use per child (including informal or free care), and we integrate out the price of childcare including free care. Both the relationship between hours of work and hours of childcare use per child and the estimated price distribution vary by age of the youngest child and number of children in the family (Brewer et al, 2005).

4 Estimating the labour supply elasticities

One key innovation in this paper is that, rather than calibrating the labour supply elasticities of various groups, we make use of labour supply elasticities derived from comparable, country-specific, structural models of labour supply. Full details of the two models used are given in Bargain et al. (2006) and Haan (2006) for Germany, and Brewer et al. (2006) for the UK, but the salient features are that both are discrete choice models of labour supply, where

[10] These micro simulation have been extensively used in previous research and are best described in Giles and McCrae, (1995) for TAXBEN, and in Steiner, et al. (2005) for STSM.

each individual *i* is assumed to choose between not working and a finite number *J* of positive hours choices, with each choice *j*=0,...*J* corresponding to a level of disposable income C_{ij}. Choice *j*=0 corresponds to not working. The attraction of this approach is that it can easily allow for non-linear and non-convex budget sets (see Blundell and MaCurdy 1999). Both models specify the direct utility function as a quadratic function in net income and hours worked. The utility is allowed to vary with observable and unobservable effects. A detailed specification for both countries can be found in the Appendix.

Labour Supply Elasticities on the Extensive and Intensive Margin

As mentioned above, we see the optimal tax model in terms of groups defined with respect to gross earnings. However, the two discrete choice labour supply models are defined with respect to (weekly) hours worked. The way that we use the structural labour supply models to calculate the intensive and extensive elasticities required by the Saez formula is described in detail in the Appendix. For the UK, elasticities are estimated from a sample of lone mothers in 2002/3, for Germany, from 2002-2004.[11] Note that the definition of the extensive elasticities given in section 2 differs from that of the conventional extensive elasticity (sometimes called the participation elasticity, or the elasticity of labour force participation), which measures the proportional increase in labour force participation in response to a 1% increase in net income in-work: for comparison with other studies, therefore, we show values of this conventional elasticity of labour force participation.[12]

Table 2 shows that the estimated elasticities differ between the countries, being generally higher in the UK. The intensive elasticities decline as weekly hours increase, but the extensive elasticities increase. That the overall labour market behaviour of lone mothers in the UK differs from those in Germany is confirmed by our estimates of the conventional elasticity of labour force participation, which stands at 1.36 for lone mothers in the UK, compared with 0.63 in Germany.

Table 2: Labour supply elasticities by working hours: UK and Germany

[11] Given this information we estimate the elasticities for the fiscal years 2001 to 2003. The tax and benefit system in Germany did hardly change during that time, so the panel dimension provides more information and variation for the analysis.

[12] In practice, we estimate this by increasing net incomes at all positive hours choices.

	Labour Supply Elasticities			
	UK		Germany	
	Extensive	Intensive	Extensive	Intensive
Part time 1	0.22	0.22	0.10	0.10
Part time 2	0.44	0.04	0.12	0.01
Part time 3	0.49	0.02	0.18	0.03
Full time 1	0.65	0.03	0.17	0.01
Full time 2	0.66	0.02	0.18	0.05
Elasticity of LFP	1.36		0.63	

Notes: : For Germany, the intervals for working hours were 0-5, 6-14, 15-21, 22-27, 28-3, 34+, with corresponding hours points 0,10,20,25,30,38. For the UK, the intervals are 0, 1-15, 16-22, 23-29, 30-36, 37+, with corresponding hours points 0,10,19,26,33,40 (the median of each band).

5 Numerical Simulation

For the numerical simulation of the optimal tax schedule we define I+1 discrete groups along the gross earnings distribution, I groups for positive earnings and in addition the group of non workers which have zero gross earnings. In the following we focus on simulations with 6 discrete groups, the non working and the working by quintiles of the positive earnings distribution. In an Appendix we provide results of simulation using deciles of the earnings distribution instead to allow for more heterogeneity. For comparative reasons we define the same income classes for Germany and the UK.[13]

Given the derived elasticities and the defined discrete earning points we can apply the Saez framework of optimal taxation to analyse optimal transfer and tax schedule for lone mothers in Germany and the UK. Therefore, we need to solve the optimal tax schedule defined above. The optimal schedule is derived subject to two constraints.

$$\sum_{0}^{I} h_i T_i = H;$$

$$\sum_{0}^{I} h_i g_i = 1.$$

[13] The income classes are defined to be deciles/ quintiles of a hypothetical earnings distribution. The hypothetical earnings distribution was constructed by assuming that each lone mother in our German sample has a 20% probability of working at the 5 positive values of hours a week, and then estimating the resulting distribution of weekly earnings. For both countries we apply the common set of cut-off points.

The first is the government's budget constraint, that is, the weighted sum of net taxes has to sum up to the budget constraint. As stressed above, for lone parents the budget constraint is negative in both countries because lone parents receive a positive net transfer financed by the rest of the society. The second constraint is a normalisation necessary for identification.

We make use of the duality of optimal income taxation framework and analyse two questions. First, we follow Bourgignon and Spadaro (2005) and derive the welfare weights assigned to the different groups along the income distribution that make the actual tax and transfer system in both countries optimal. Second, assuming a specific welfare function we design the optimal tax and transfer system for lone mothers in Germany and the UK.

5.1 Optimal Weights

In an application for France, Bourgignon and Spadaro (2005) invert the Mirrlees model and find that, if intensive elasticities are low (compared to those we have estimated for Germany and the UK), then the French tax and transfer schedule is optimal under a Paretian government. However, when they assume higher elasticities, they show that the actual French tax and transfer system is only optimal if the authority imputes negative social welfare weights to individuals at the upper end of the income distribution. We follow this approach and derive the weights that make the given tax and transfers system in Germany and the UK optimal using the estimated labour supply elasticities along the extensive and the intensive margin.

Table 3: Optimal weights for the taxation of lone mothers. UK versus Germany

	Gross Earnings	Net Income	Net Tax	Marginal Tax Rate	Share	Intensive Elasticity	Extensive Elasticity	Opt. Weights	Relative Opt. Weights
					United Kingdom				
0	0	274.78	-274.78	-	0.48	-	-	1.65	1.00
1	100.08	318.69	-218.61	0.56	0.12	0.14	0.32	0.43	0.26
2	190.24	367.91	-177.67	0.45	0.11	0.03	0.55	0.43	0.26
3	261.58	399.60	-138.03	0.56	0.09	0.02	0.64	0.30	0.18
4	343.88	435.99	-92.11	0.56	0.09	0.03	0.61	0.33	0.20
5	530.19	522.84	7.35	0.53	0.10	0.04	0.45	0.44	0.27
					Germany				
0	0	244.54	-244.54	-	0.29	-	-	1.73	1.00
1	108.40	297.08	-188.68	0.52	0.08	0.10	0.13	0.79	0.46
2	192.63	328.39	-135.75	0.63	0.06	0.03	0.17	0.79	0.46
3	264.39	351.15	-86.75	0.68	0.12	0.01	0.22	0.70	0.41
4	347.94	386.07	-38.13	0.58	0.15	0.03	0.27	0.63	0.36
5	553.54	488.53	65.00	0.50	0.31	0.03	0.21	0.77	0.45

Notes: Cut off points for the positive earnings points (in €): 153, 228, 300, 405. All income and tax information are the mean average values per week. Marginal tax rate is calculated as change in net tax over change in gross earnings between adjacent groups.
Source: SOEP and FRS.

Table 4 shows for each group mean net tax payments, mean net income, average marginal tax rates, mean elasticities, and the actual share of the population located in each band.[14]

The share of lone mothers at the discrete earnings points differs markedly between Germany and the UK. As shown in the previous section, almost half of the lone mothers in the UK are located at zero gross earnings. The distribution over positive earnings is fairly even, with about 10% at each point. In contrast, in Germany, only less than one third of lone mothers have zero earnings, about 40% are at the low to middle earnings points, and the remaining lone mothers (about one third) are at the top quintile. The higher labour market participation, higher hours of work given labour market participation, and higher hourly wages together all mean that average gross earnings are considerably higher in Germany than in the UK. The UK has a more generous transfer system towards lone mothers than Germany. At every earnings point net transfer are higher in the UK. The generosity even leads to a higher net household income at every point in the UK despite the described gross earnings gap in favour of Germany. As shown in column 3, the transfer system on average does not give larger benefits to the working poor than to non-workers. That implies marginal tax rates are generally non-negative. Thus, in the current tax and benefit system of both countries in-work credits are not implemented. For Germany this finding is not surprising as in the tax and transfer system implemented in 2002 no substantial transfers are conditioned on working.[15] However, as stressed above, the current British tax system conditions some transfers on working, the WTC. In general, in the 2002/3 transfer system, low-wage part-time workers could receive higher net transfers in-work than if they did not work, but only if they had two or more children, and – crucially – only if they would not receive housing benefit or council tax benefit if they did not work. In practice, the vast majority of non-working lone parents receive at least one of these. Lastly, there are important differences in the estimated labour supply elasticities along the discrete earnings points. We find that in both countries elasticities

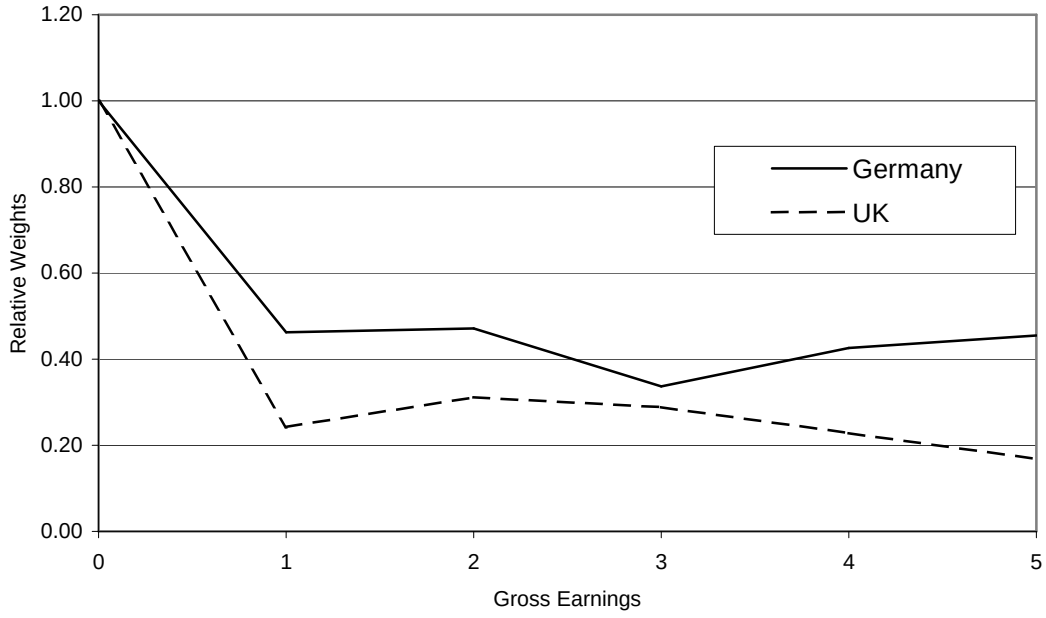
[14] As the Appendix sets out, the mean net tax, disposable income and elasticities shown in Table 4 are over the whole sample, not just those lone mothers who are observed to have gross earnings in each band. This is because we are able to estimate elasticities for each individual at each discrete band, and we can calculate net taxes for every individual for any level of gross earnings.

[15] In course of the Hartz reform in 2005 a child supplement has been introduced that is conditioned on working. However, the supplement is so minor that the structure of the German transfers system has been not affected.

on the extensive margin exceed the elasticity on the intensive margin. In the UK however, this difference is far more pronounced than in Germany.

The weights under which the current UK and German tax and transfer system for lone mothers are optimal, given our estimated labour supply elasticities, are presented in the last two columns of table 4 and graphically presented in figure 2. To anchor the social welfare weights, Saez (2002) requires that the sum of weights, weighted by the share of the population that choose each band of earnings, is equal to one. This scaling, though, makes it difficult to compare the weights estimates for two countries with such different patterns of work. To provide a better cross-country comparison, we show the derived optimal weights expressed relative to the weight given to the non-workers.

Figure 2: Optimal weights by gross earning groups: UK and Germany



In general, we find that both countries' tax and transfer systems are optimal only if the government has strong concern for redistributing to non-workers: the weights for non-working women are relatively high, and those for working women are low, and decline by little as earnings rise or are constant across positive earnings. The results for the UK show that the government assigns weights to the working population which are about 20% of the weights for non working lone mothers. In Germany, working lone mothers have slightly higher weights which are about 40% of the non-working lone mothers. Our findings imply

that in both Germany and the UK, the government has stronger preferences for redistribution to the non-workers, yet this preference for redistribution is higher in the UK than it is in Germany. It is worth considering how this result arrives: it is driven by the relatively high elasticities on the extensive margin. This implies that a shift in the tax burden from the working poor to the non-workers would induce a relatively large numbers of non-working lone mothers to start working because extensive elasticities are high. On the other hand, this would not have a strong negative impact on the labour supply of those already in-work because intensive elasticities are low. However, as discussed above, in Germany and the UK transfers to the non-working are higher than to the working poor. Thus the only way that the design of the current tax and transfer system is optimal is by assigning a much higher weight to the non-workers than to the working poor. In Germany, extensive elasticities are relatively lower than those in the UK and therefore the current system in Germany is found to be optimal with a less strong redistributive taste to the non-working lone mothers.

Optimal Weights by age of children

As we have shown in Table 1, the working behaviour of lone mothers markedly differs by the age of the youngest child. In both countries, we find that participation rates are very low for lone mothers with pre-school children. Moreover, form a normative point of view, there exist arguments that a government should provide high out of work transfers for women with pre-school children so that they can afford to care for their children during early childhood. On the contrary, as this is one of the groups with the lowest participation rates, making work pay policies should be promising amongst this group. Therefore, we derive optimal welfare weights separately for lone mothers with and without school age children. We calculate the weights separately for each group, treating the taxation of the rest of the lone mothers as exogenous.

Table 4a: Optimal weights for the taxation of lone mothers by age of children: Germany

	Gross Earnings	Net Income	Net Tax	Marginal Tax Rate	Share	Intensive Elasticity	Extensive Elasticity	Opt. Weights	Relative Opt. Weights
Lone mothers with children younger school age									
0	0	277.49	-277.49	-	0.54	-	-	1.38	1.00
1	103.88	300.51	-196.63	0.78	0.09	0.05	0.07	0.60	0.43
2	193.26	338.57	-145.31	0.57	0.07	0.01	0.18	0.61	0.44
3	263.12	348.24	-85.12	0.86	0.10	0.01	0.25	0.35	0.25
4	345.51	370.84	-25.33	0.73	0.07	0.02	0.14	0.58	0.42
5	544.29	444.61	99.68	0.63	0.12	0.01	0.16	0.63	0.46
Lone mothers with school age children									
0	0	230.45	-230.45	-	0.19	-	-	2.03	1.00

1	110.82	286.60	-175.78	0.49	0.08	0.11	0.14	0.82	0.41
2	192.25	309.50	-117.25	0.72	0.06	0.04	0.16	0.79	0.39
3	264.98	334.00	-69.02	0.66	0.13	0.03	0.23	0.64	0.32
4	348.87	373.70	-24.83	0.53	0.17	0.04	0.21	0.75	0.37
5	555.65	485.02	70.63	0.46	0.38	0.05	0.14	0.79	0.39

Notes: Cut off points for the positive earnings points (in €): 153, 228, 300, 405. All income and tax information are the mean average values per week. Marginal tax rate is calculated as change in net tax over change in gross earnings between adjacent groups.
Source: SOEP.

For Germany we find the expected pattern of average net taxes, net household income, and of the shares at the discrete earnings points by the age of the youngest child. Lone mothers with children below school age tend to receive higher transfers and are more likely not to work. The distribution along the earnings distribution for lone mothers with school-aged children however is rather different. More than one third of this group is located at the highest quintile of the earnings distribution and less than 20% are not working. Despite these differences we find that the labour supply behaviour between both groups is rather similar. Along the extensive margin, we find relative higher elasticities compared to those on the intensive margin.

Turning to the weights the government assigns to each discrete group, our results show a very similar patter for both groups. To make the current system in Germany optimal, the government reveals higher preferences for the non-working relative to the working lone mothers. On average the government assigns about 40% of the weight for the non working to the working population.

Table 4b: Optimal weights for the taxation of lone mothers by age of children: Britain

	Gross Earnings	Net Income	Net Tax	Marginal Tax Rate	Share	Intensive Elasticity	Extensive Elasticity	Opt. Weights	Relative Opt. Weights
				Lone mothers with children younger school age					
0	0	281.73	-281.73	-	0.67	-	-	1.44	1.00
1	98.12	319.70	-221.58	0.61	0.13	0.15	0.41	0.12	0.09
2	189.52	371.28	-181.76	0.44	0.08	0.02	0.72	0.19	0.13
3	260.80	403.33	-142.53	0.55	0.04	0.02	0.85	0.02	0.02
4	342.68	440.28	-97.60	0.55	0.04	0.02	0.86	0.01	0.01
5	511.50	523.58	-12.08	0.51	0.04	0.03	0.79	0.09	0.06
				Lone mothers with school age children					
0	0	271.05	-271.05	-	0.38	-	-	1.80	1.00
1	101.43	317.99	-216.56	0.54	0.12	0.13	0.27	0.57	0.32
2	190.65	366.00	-175.34	0.46	0.13	0.03	0.47	0.53	0.30
3	261.94	397.85	-135.91	0.55	0.11	0.03	0.55	0.42	0.23
4	344.36	434.29	-89.93	0.56	0.12	0.03	0.52	0.44	0.25
5	535.79	522.62	13.18	0.54	0.14	0.04	0.35	0.55	0.31

Notes: Cut off points for the positive earnings points (in €): 153, 228, 300, 405. All income and tax information are the mean average values per week. Marginal tax rate is calculated as change in net tax over change in gross earnings between adjacent groups.
Source: FRS.

In the UK the difference in the redistributive taste of the government for lone mothers with and without school age children is more pronounced. Relative to the non-working lone mothers the government assigns very low weights to the working lone mothers with the youngest child younger school age. The results suggest that the government basically does not care for this group at all as the weights are close to zero. The weight for a lone mother with a school aged child is at about 30% of the weight the government assigns to the same women not working. Again, these results are driven by the relative higher elasticities on the extensive margin, and the large share of non-working lone mothers with pre-school child.

5.2 Optimal tax schedule

As discussed in the previous section, neither in the UK nor in Germany, the tax and transfer system has negative marginal tax rates. However, as shown by Saez (2002) negative marginal tax rates can become optimal when extensive elasticities are relatively important compared to intensive elasticities. It is therefore of interest to find out under what social welfare functions would increased transfers to the working poor become optimal. Recall that rationalising the current transfer system in both countries requires the government to have relatively strong desires to redistribute to non-working lone mothers.

We therefore derive the optimal tax schedule across the gross earnings points under a class of social welfare weights, g_i, that decrease with gross earnings as follows:

$$g_i = \frac{1}{\exp(\tilde{y}_i)^v - k},$$

where $\tilde{y}$ is the gross earnings at point *i* relative to the gross earnings at the highest earnings point, k is a shifting parameter. The redistributive taste of the government is expressed with *v*: the higher *v,* the higher is the redistributive taste, and we provide three scenarios with varying taste for redistribution: a scenario with low redistributive taste, v=0.5, medium taste v=1, and high redistributive taste v=2. [16] As in the tables above, we present the weights in absolute and

[16] We have experimented with several function forms of a welfare function decreasing with gross earnings. The results are robust to the choice of the functional form. For these results we have chosen k =0.25. More extreme taste parameters v=0.1 and v=4 yield the expected results.

in relative (i.e., scaled to the weight given to the non-workers) to provide a better country comparison.

Table 5: Optimal tax rates for lone mothers. UK versus Germany

Gross Earnings	Net Tax	Opt. Weights	Relative Weight	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax
Britain												
	Status quo				V=0.5			V=1			v=2	
0.00	-274.78	1.65	1.00	1.33	1.00	-215.04	1.33	1.00	-252.06	1.33	1.00	-274.88
100.08	-218.61	0.43	0.26	1.17	0.88	-268.35	1.03	0.78	-254.88	0.81	0.61	-242.66
190.24	-177.67	0.43	0.26	1.06	0.80	-237.54	0.86	0.64	-211.93	0.57	0.43	-191.71
261.58	-138.03	0.30	0.18	0.98	0.74	-201.58	0.73	0.55	-168.84	0.43	0.32	-146.78
343.88	-92.11	0.33	0.20	0.89	0.67	-152.81	0.62	0.46	-109.93	0.31	0.23	-85.53
530.19	7.35	0.44	0.27	0.71	0.54	-39.20	0.41	0.30	24.72	0.14	0.11	54.64
Germany												
	Status quo				V=0.5			V=1			v=2	
0.00	-244.54	1.68	1.02	1.33	1.00	-206.49	1.33	1.00	-275.59	1.33	1.00	-299.96
108.40	-183.04	0.78	0.47	1.17	0.88	-256.47	1.03	0.78	-239.87	0.81	0.61	-242.10
192.63	-127.96	0.79	0.48	1.06	0.80	-183.60	0.86	0.64	-165.53	0.57	0.43	-185.43
264.39	-74.07	0.57	0.34	0.98	0.74	-120.78	0.73	0.55	-101.94	0.43	0.32	-89.05
347.94	-24.97	0.72	0.43	0.89	0.67	-49.54	0.62	0.46	-26.98	0.31	0.23	-14.71
553.54	76.04	0.77	0.46	0.71	0.54	100.92	0.41	0.30	139.48	0.14	0.11	156.01

Notes: Cut off points for the positive earnings points (in €): 153, 228, 300, 405. All income and tax information are the mean average values per week. Source: SOEP 2001-2003 and FRS 2002/3.

Assuming a low redistributive taste, in-work credits with negative marginal tax rates become optimal in both countries. In this welfare scenario, the optimal design in the UK would imply that transfers are reduced for the non-working lone mothers, while for all working lone mother transfers increase compared to the status quo system in 2002/3. The tax credit would be of remarkable size: lone mothers in the first two positive earnings groups would receive an in-work credit, with higher net transfers than the non-working lone mothers. A similar result holds for Germany. When the government has a low taste of redistribution, it is optimal to transfer higher benefits to the working poor, that is to those lone mothers earning at the lowest quintile of the gross earnings distribution, than to lone mothers out of work. For lone mothers above the lowest quintile in-work credits are not optimal, yet in comparison to the status quo, transfers increase for all working except for those in the highest quintile. Thus in Germany,

the higher transfers for the working would be financed by the lone mothers out of work, and by those earning in the top quintile.

In a scenario with medium redistributive taste, in the UK, it is optimal to tax the non working and the working at the lowest discrete groups at roughly the same rate, -252 Euro per week for the non working and –255 Euro for the working.. That implies in-work credits s are just optimal. Allowing for more discrete groups (Appendix) we show that larger in-work credits for discrete groups with lower average positive earnings become optimal in this redistributive scenario. In contrast, for Germany we find that in this scenario a tax system with only positive marginal tax rate, i.e. without in-work credits is optimal. With more discrete groups having lower average gross earnings we show that for the lowest two groups small in-work credits are optimal in this scenario.

In both countries the optimal tax schedule does not contain negative marginal tax rates when we assume high redistributive taste of the government. Net taxes are monotonously increasing with gross earnings. In Britain, however tax rates increase at a lower rate than in Germany. That implies the working poor are less heavily taxed relative to the non working than in Germany. This results hold regardless of the number of discrete groups, when simulating the tax system for 10 discrete groups, we find the same pattern.

In general, our findings indicate that is more optimal to design in-work credits in the British tax and benefit system. Even with medium taste of redistribution we find that in-work credits are optimal which. As mentioned above, this result is mainly driven by the higher extensive elasticities in Britain leading to higher positive labour supply responses on the extensive margin.

Optimal tax schedule by age of children

In the following we derive the optimal tax schedule for lone mothers by age of the youngest child. This analysis is based on the assumption that the government conditions taxation not only on gross earnings but as well on the age of the youngest child. Again, when deriving the optimal tax and transfer system for a subgroup of lone mothers we hold taxation of the rest of the population constant.

Table 6a: Optimal tax rates for lone mothers. by age of children: Germany

Gross Earnings	Net Tax	Opt. Weights	Relative Weight	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax
Mothers with children younger school age												
		Status quo			**v=0.5**			**v=1**			**v=2**	
0.00	**-277.49**	**1.38**	**1.00**	**1.33**	**1.00**	**-212.15**	**1.33**	**1.00**	**-267.91**	**1.33**	**1.00**	**-287.39**
103.88	**-196.63**	**0.60**	**0.43**	**1.17**	**0.88**	**-329.48**	**1.03**	**0.78**	**-257.78**	**0.81**	**0.61**	**-222.02**
193.26	**-145.31**	**0.61**	**0.44**	**1.06**	**0.80**	**-244.30**	**0.86**	**0.64**	**-177.61**	**0.57**	**0.43**	**-160.45**
263.12	**-85.12**	**0.35**	**0.25**	**0.98**	**0.74**	**-175.59**	**0.73**	**0.55**	**-110.27**	**0.43**	**0.32**	**-87.55**
345.51	**-25.33**	**0.58**	**0.42**	**0.89**	**0.67**	**-101.15**	**0.62**	**0.46**	**-34.90**	**0.31**	**0.23**	**-11.89**
544.29	**99.68**	**0.63**	**0.46**	**0.71**	**0.54**	**84.10**	**0.41**	**0.30**	**148.47**	**0.14**	**0.11**	**167.29**
Mother with school age children												
		Status quo			v=0.5			v=1			v=2	
0.00	**-230.45**	**2.03**	**1.00**	**1.33**	**1.00**	**-201.72**	**1.33**	**1.00**	**-279.81**	**1.33**	**1.00**	**-309.19**
110.82	**-175.78**	**0.82**	**0.41**	**1.17**	**0.88**	**-243.55**	**1.03**	**0.78**	**-239.82**	**0.81**	**0.61**	**-238.55**
192.25	**-117.25**	**0.79**	**0.39**	**1.06**	**0.80**	**-180.85**	**0.86**	**0.64**	**-172.41**	**0.57**	**0.43**	**-168.55**
264.98	**-69.02**	**0.64**	**0.32**	**0.98**	**0.74**	**-117.87**	**0.73**	**0.55**	**-106.91**	**0.43**	**0.32**	**-101.87**
348.87	**-24.83**	**0.75**	**0.37**	**0.89**	**0.67**	**-50.82**	**0.62**	**0.46**	**-38.51**	**0.31**	**0.23**	**-33.99**
555.65	**70.63**	**0.79**	**0.39**	**0.71**	**0.54**	**108.14**	**0.41**	**0.30**	**135.35**	**0.14**	**0.11**	**145.26**

Notes: Cut off points for the positive earnings points (in €): 153, 228, 300, 405. All income and tax information are the mean average values per week.
Source: SOEP

Assuming that the government has a low distributive taste, in Germany in-work credits are optimal regardless of the age of the child. Yet, the design of the tax credits differs by the age of the youngest child. In-work transfers for lone mothers with pre-school children are more generous than for working single mothers with older children. It is optimal for the government to provide in-work transfers towards the working at the first two quintiles and the transfers are of substantial size. In contrast, for lone mothers with older children in-work credits are only optimal at the first earnings point, and the credit is markedly lower for this group. Assuming a medium or high taste for redistribution, the results suggest that in-work credits for neither of the groups are optimal. In general, the findings suggest that in Germany it is more optimal to design in-work credits for lone mothers with pre-school children. This result is mainly driven by the low participation rate in this group and hence making work pay policies affect a large part of the relevant population.

Table 6a: Optimal tax rates for lone mothers. by age of children: Britain

Gross Earnings	Net Tax	Opt. Weights	Relative Weight	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax
Mothers with children younger school age												
		Status quo			**v=0.5**			**v=1**			v=2	
0.00	**-281.73**	**1.44**	**1.00**	**1.33**	**1.00**	**-241.90**	**1.33**	**1.00**	-259.10	1.33	1.00	-271.84
98.12	**-221.58**	**0.12**	**0.09**	**1.17**	**0.88**	**-283.65**	**1.03**	**0.78**	-263.70	0.81	0.61	-245.92
189.52	**-181.76**	**0.19**	**0.13**	**1.06**	**0.80**	**-261.15**	**0.86**	**0.64**	-228.82	0.57	0.43	-201.96
260.80	**-142.53**	**0.02**	**0.02**	**0.98**	**0.74**	**-235.29**	**0.73**	**0.55**	-195.84	0.43	0.32	-165.93
342.68	**-97.60**	**0.01**	**0.01**	**0.89**	**0.67**	**-200.91**	**0.62**	**0.46**	-150.54	0.31	0.23	-117.11
511.50	**-12.08**	**0.09**	**0.06**	**0.71**	**0.54**	**-116.25**	**0.41**	**0.30**	-48.59	0.14	0.11	-12.23
Mother with school age children												
		Status quo			**v=0.5**			**v=1**			v=2	
0.00	**-271.05**	**1.80**	**1.00**	**1.33**	**1.00**	**-203.63**	**1.33**	**1.00**	-253.58	1.33	1.00	-281.70
101.43	**-216.56**	**0.57**	**0.32**	**1.17**	**0.88**	**-265.86**	**1.03**	**0.78**	-253.69	0.81	0.61	-244.04
190.65	**-175.34**	**0.53**	**0.30**	**1.06**	**0.80**	**-227.34**	**0.86**	**0.64**	-205.84	0.57	0.43	-189.75
261.94	**-135.91**	**0.42**	**0.23**	**0.98**	**0.74**	**-185.37**	**0.73**	**0.55**	-158.49	0.43	0.32	-141.50
344.36	**-89.93**	**0.44**	**0.25**	**0.89**	**0.67**	**-130.86**	**0.62**	**0.46**	-95.45	0.31	0.23	-76.55
535.79	**13.18**	**0.55**	**0.31**	**0.71**	**0.54**	**-6.45**	**0.41**	**0.30**	49.65	0.14	0.11	74.36

For Britain, we find a similar picture, yet differences are even stronger when the government designs the tax and transfer system differently by the age of the youngest child. In the scenario with a low redistributive taste, in-work credits are optimal for lone mothers regardless of the age of the youngest child. As transfers for lone mother with younger children are in general more generous, the size of in-work credits is larger for this group. However, relative to the transfers to the non working, our results suggest that in-work credits for lone mothers with school age children are more generous. Assuming a medium redistributive taste of the government, we find that a small credit for lone mothers with pre school age children is optimal, yet not for the single mothers with older children. In the last scenario, we find that similar to the status quo tax and transfer system it is optimal to provide highest transfers to the non working lone mothers which are decreasing with gross earnings.

6. Conclusion

In this paper we apply the optimal tax rule suggested by Saez (2002) to empirically discuss the optimal tax and transfer design in Germany and the UK. The key advance of this paper is that we combine the theoretical model with a structural estimation of households` labour supply. Thus we are able to allow for heterogeneity between groups regarding their behaviour adjustment rather than calibrating an overall labour supply elasticity for the whole society.

When focusing on lone parents we have shown that in-work credits for this group are optimal from a social welfare perspective with relatively low and medium taste for redistribution in both Germany and the UK. Even with a high taste for distribution it is optimal in the UK to tax the non working and the poorest working women at the same rate. These results are driven by relatively high elasticities on the extensive margin which implys a high positive participation response of the non working.

By the same token we show that the given tax schedules in both countries, without an explicit in-work credit, are only optimal if the government has a high welfare value for the non working lone mothers and a relatively low taste for redistribution towards the working lone mothers.

These findings have been derived with respect to a specific group, lone mothers, as in the current political debate this is the main target group for in-work credits. However, the main findings of this analysis might carry over to other groups or even to the whole population. As mentioned above, so far the optimal tax literature has not developed a theoretical framework incorporating the joint decision of households that can be empirically analysed. However, as we have shown, when elasticities on the extensive margin are relatively high relative to the potential negative reactions on the intensive margin, labour supply effects of in-work credits will be positive and depending on the distributive taste of the government are optimal.

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Appendix 1: Data and descriptive statistics

The database used for Germany is the German Socio Economic Panel (SOEP), a representative sample of over 12,000 households living in Germany interviewed annually. (Haisken De-New and Frick, 2001) For the empirical analysis, an unbalanced panel for the years 2001 - 2003 is used. The population consists of lone women with at least one dependent child that are aged between 20 and 60 years. Excluded are adults in full-time education, the self-employed or retired, and households with missing information, leaving 1,009 lone mothers.

According to the empirical distribution of working hours we have chosen 6 discrete working hours alternative, inactivity three part time and two full time working alternatives. The following table yields descriptive statistics about the variables that enter the estimation. Alternative specific variables are listed by working hours.

Table A1: Working hours of lone mothers

	Germany			Britain		
	Working hours	Share	Net income	Working hours	Share	Net income
Inactivity	0	0.29	1049	0	0.55	1200
Part time 1	10	0.06	1308	10	0.06	1341
Part time 1	20	0.11	1436	19	0.11	1558
Part time 3	25	0.07	1569	26	0.05	1639
Full time 1	30	0.13	1655	33	0.07	1785
Full time 2	40	0.34	1856	40	0.15	1864

Notes: Germany: the following intervals for working hours have been chosen 0-5, 5-15, 15-22, 22-28, 28-35, >=35. UK: the following intervals for working hours have been chosen: 0.1-15, 16-23, 24-29, 30-36, =35. Source: authors' calculations from FRS, 2002/3: see Brewer et al (2005) for precise details of sample selection.

Table A2: Descriptive Statistics

	Germany	Britain
Age	39.02	35.04
Share with children younger 3	0.11	0.20
Share with children older 3 and younger 7	0.20	0.28
Share living in East Germany	.25	-
Share with a low educational Degree	.35	0.69
Share with a medium educational Degree	.50	0.27
Share with a high educational Degree	.15	0.04
Number of observations	1009	1881

Notes: Germany: Low education: no degree, or Hauptschule, high education: Abitur or Fachabitur, medium eduction. rest. UK: Low education is defined as ceasing full-time education at or before the age of 16; high education degree means ceasing full-time education aged 21 or older.

Appendix 2: Discrete Choice Labour Supply Estimation

Discrete choice models of labour supply are based on the assumption that a household can choose among a finite number J+1 of working hours (J positive hours points and non-employment); each hour j=0,…,J corresponds to a given level of disposable income Cij and each discrete bundle of leisure and income provides a different level of utility. The utility Vij derived by household i from making choice j is assumed to depend on a function U of the woman's leisure term Lfij, her disposable income Cij and household characteristics Zi, and on a random term εij. When the error term εij is assumed to be identically and independently distributed across alternatives and households according to the Extreme Value distribution, McFadden (1974) proves that the probability that alternative k is chosen by household i is given by:

$$\Pr_{ik} = \frac{\exp(V_{ik})}{\sum_{j=0}^{J} \exp(V_{ij})}, k \in J$$

The likelihood for a sample of observed choices can be derived from that expression and maximised to estimate the parameters of function *U*. We assume a quadratic specification of the utility function as in Blundell et al. (2000). In the estimation we include observed and unobserved heterogeneity by allowing income and leisure to vary with observed and unobserved characteristics. The specifications slightly differ between Germany and the UK such that country specificities and the differences in the data structures, e.g. the panel structure of the SOEP can be accounted for.

For Germany, the specification to be estimated is similar as in Bargain et al. (2006) or Haan (2006) and has the following form:

$$V_{ijt} = \alpha_1 y_{i\,jt} + \alpha_2 l_{ijt} + \alpha_3 y^2{}_{ijt} + \alpha_4 l^2{}_{ijt} + \alpha_5 y l_{ijt}$$

where the marginal utility of income and leisure varies by age, education, number and age of children, region, health status, nationality. To capture the disutility related to flexible arrangements, we follow van Soest (1995) and include dummy variables for the part time categories. More over the leisure time of the women differs by unobservable effects which are modelled non parametrically following Heckman and Singer (1984). We assume a discrete distribution with two (k) mass points :

$$a_1 = \beta_1 + \gamma_1 X_{1it}$$

$$a_2 = \beta_2 + \gamma_2 X_{2it} + \mu_k, k \in \{1, 2\}.$$

For the UK, the specification and parameter estimates are set out in full in Brewer et al (2005). As in the model for Germany, the utility function is quadratic in hours of work and income, but unlike the model for Germany, all alpha coefficients are functions of observed heterogeneity (age and education of mother, number of children, age of youngest child, ethnicity and region of residence), and the linear coefficients on income and leisure also vary with unobserved heterogeneity.

Appendix 3: Labour supply elasticities by gross earnings

The labour supply elasticities are derived numerically based on the estimated preferences of the labour supply model. Recall that Saez (2002) formula for the optimal tax is written in terms of intensive and extensive elasticities respectively defined as:

$$\mu_i = \frac{C_i - C_{i-1}}{h_i} \frac{dh_i}{d(C_i - C_{i-1})};$$

and:

$$\eta_i = \frac{C_i - C_0}{h_i} \frac{dh_i}{d(C_i - C_0)};$$

where such elasticities are implicitly averages across the relevant population, and $i = 0…J$ indexes the choice ($i=0$ corresponds to not working).

To use this model to say something about the optimal tax function in practice requires us to view the different groups as different groups defined with respect to gross earnings (just as Saez (2002) does in his numerical example). For each individual k in our sample, we therefore estimate the elasticities $\tilde{\mu}_{i,k}$ and $\tilde{\eta}_{i,k}$, where $i = 0…J$ indexes the hours choice.[17] By definition, the intensive and extensive elasticity are identical for $i=1$ (the first choice of positive hours worked).

We then translate these elasticities in terms of weekly hours worked into elasticities in terms of gross weekly earnings by calculating:

[17] We are able to estimate an elasticity for each individual by taking repeated draws from the extreme value errors, and calculating (for example) the fraction of times a given individual's preferred choice would change from choice i to choice $i-1$ in response to a 1% change in $C_i - C_{i-1}$ divided by the fraction of times the individual's preferred choice is choice i (and equivalently for the extensive elasticities). See 5.2.7 in Creedy et al (2002). When estimating the elasticities using the labour supply model for the UK and Germany, we assume full take-up of (complete program participation in) all benefits and tax credits.

$$\mu_i = \sum_{\forall k: H_i . w_k \in Y_i} \overline{\widetilde{\mu}_{i,k}},$$

(and equivalently for the extensive elasticity), where the bar denotes the mean, w_k is the (actual or predicted) hourly wage for each individual, $H_i.w_k$ measures gross (weekly) earnings for individual k at choice i, and the set of Y_i defines intervals of gross earnings, and i=1…J (where J = 5 or 10) indexes the intervals of gross earnings.[18]

Appendix 4: Optimal weights for the taxation of lone mothers by earnings deciles : UK versus Germany

	Gross Earnings	Net Income	Net Tax	Marginal Tax Rate	Share	Intensive Elasticity	Extensive Elasticity	Opt. Weights	Relative Opt. Weights
					United Kingdom				
0	0.00	274.78	-274.78		0.48	0.00	0.00	1.64	1.00
1	76.25	305.75	-229.49	0.59	0.05	0.20	0.26	0.40	0.24
2	130.81	335.38	-204.58	0.46	0.07	0.06	0.40	0.51	0.31
3	173.26	359.64	-186.38	0.43	0.06	0.03	0.50	0.47	0.29
4	210.55	377.80	-167.25	0.51	0.05	0.02	0.61	0.37	0.23
5	245.79	392.23	-146.44	0.59	0.05	0.02	0.66	0.27	0.17
6	281.73	409.01	-127.28	0.53	0.04	0.03	0.62	0.33	0.20
7	320.46	425.26	-104.80	0.58	0.04	0.02	0.63	0.29	0.18
8	371.33	448.56	-77.23	0.54	0.05	0.03	0.58	0.36	0.22
9	446.10	477.30	-31.20	0.62	0.05	0.03	0.52	0.37	0.23
10	642.02	583.40	58.62	0.46	0.05	0.05	0.36	0.57	0.35
					Germany				
0	0.00	244.54	-244.54		0.29	0.00	0.00	1.66	1.00
1	86.00	294.98	-202.59	0.49	0.05	0.12	0.12	1.07	0.64
2	129.84	299.09	-164.34	0.87	0.03	0.08	0.14	0.43	0.26
3	173.68	320.02	-139.91	0.56	0.02	0.05	0.13	0.82	0.50
4	211.04	336.52	-116.36	0.63	0.04	0.01	0.20	1.13	0.68
5	246.44	343.98	-82.63	0.95	0.05	0.01	0.20	0.22	0.14
6	282.22	358.27	-65.58	0.48	0.07	0.01	0.24	0.55	0.33
7	321.93	380.23	-42.94	0.57	0.06	0.04	0.31	0.72	0.43
8	373.03	391.70	-7.63	0.69	0.08	0.03	0.22	0.70	0.42
9	447.39	430.04	28.41	0.48	0.11	0.04	0.29	0.68	0.41
10	659.19	546.76	123.45	0.45	0.20	0.03	0.13	0.86	0.52

Notes: Cut off points for the positive earnings points (in €): 107, 153, 193, 228, 264, 300, 344, 405,and 502. All income and tax information are the mean average values per week. Marginal tax rate is calculated as change in net tax over change in gross earnings between adjacent groups.
Source: SOEP and FRS.

[18] One drawback from having to perform this translation from elasticities defined wrt hours worked to elasticities defined wrt gross earnings is that it is not the case that the estimated intensive elasticity is identical to the estimated extensive elasticity in the first gross earnings interval.

Appendix 5: Optimal taxation of lone mothers by earnings deciles : UK versus Germany

Gross Earnings	Net Tax	Opt. Weights	Relative Weight	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax
Britain												
	Status quo				V=0.5			V=1			v=2	
0.00	-274.78	1.64	1.00	1.33	1.00	-203.06	1.33	1.00	-245.02	1.33	1.00	-271.78
76.25	-229.49	0.40	0.24	1.22	0.92	-275.18	1.12	0.84	-264.69	0.95	0.72	-256.02
130.81	-204.58	0.51	0.31	1.17	0.88	-278.57	1.03	0.78	-252.64	0.81	0.61	-231.75
173.26	-186.38	0.47	0.29	1.12	0.84	-259.15	0.95	0.71	-231.63	0.69	0.52	-208.06
210.55	-167.25	0.37	0.23	1.08	0.81	-238.87	0.89	0.67	-211.64	0.61	0.46	-188.44
245.79	-146.44	0.27	0.17	1.05	0.78	-222.32	0.83	0.62	-191.08	0.54	0.40	-167.25
281.73	-127.28	0.33	0.20	1.01	0.76	-203.95	0.78	0.58	-166.67	0.48	0.36	-141.01
320.46	-104.80	0.29	0.18	0.97	0.73	-181.4	0.72	0.54	-137.89	0.42	0.31	-111.14
371.33	-77.23	0.36	0.22	0.93	0.70	-151.04	0.66	0.50	-99.07	0.35	0.26	-70.18
446.10	-31.20	0.37	0.23	0.87	0.65	-99.642	0.58	0.44	-38.65	0.28	0.21	-7.51
642.02	58.62	0.57	0.35	0.71	0.54	26.213	0.41	0.30	108.01	0.14	0.11	146.53
Germany												
	Status quo				V=0.5			V=1			v=2	
0.00	-244.54	1.66	1.00	1.33	1.00	-186.60	1.33	1.00	-268.70	1.33	1.00	-304.08
86.00	-202.59	1.07	0.64	1.22	0.92	-304.96	1.12	0.84	-304.60	0.95	0.72	-266.61
129.84	-164.34	0.43	0.26	1.17	0.88	-273.90	1.03	0.78	-278.72	0.81	0.61	-233.68
173.68	-139.91	0.82	0.50	1.12	0.84	-233.17	0.95	0.71	-241.87	0.69	0.52	-196.04
211.04	-116.36	1.13	0.68	1.08	0.81	-197.86	0.89	0.67	-211.12	0.61	0.46	-168.51
246.44	-82.63	0.22	0.14	1.05	0.78	-164.83	0.83	0.62	-188.10	0.54	0.40	-145.99
282.22	-65.58	0.55	0.33	1.01	0.76	-131.11	0.78	0.58	-85.82	0.48	0.36	-86.336
321.93	-42.94	0.72	0.43	0.97	0.73	-94.18	0.72	0.54	-49.66	0.42	0.31	-49.287
373.03	-7.63	0.70	0.42	0.93	0.70	-48.18	0.66	0.50	-3.40	0.35	0.26	-1.9297
447.39	28.41	0.68	0.41	0.87	0.65	13.32	0.58	0.44	59.75	0.28	0.21	63.135
659.19	123.45	0.86	0.52	0.71	0.54	195.33	0.41	0.30	250.52	0.14	0.11	258.3

Notes: Cut off points for the positive earnings points (in €): 107, 153, 193, 228, 264, 300, 344, 405, and 502. Source: SOEP 2001-2003 and FRS 2002/3.