

On Income Distribution, Incentive Effects and Optimal Income Taxation Author(s): Efraim Sadka Source: *The Review of Economic Studies*, Vol. 43, No. 2, (Jun., 1976), pp. 261–267 Published by: The Review of Economic Studies Ltd. Stable URL: <u>http://www.jstor.org/stable/2297322</u> Accessed: 28/04/2008 13:16

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On Income Distribution, Incentive Effects and Optimal Income Taxation

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Recent theories of optimal income taxation consider two alternative social ordering relations: the classical sum-of-utilities (i.e. an additive social welfare function) and Rawls' max-min criterion. The optimal tax under either of these criteria is analysed in two different models. In the first, which was introduced by Mirrlees [2] and which will be referred to as the labour model, the income of an individual depends on the number of hours he works. In the second, which was introduced by Sheshinski [6] and which will be referred to as the education model, it is the level of education which determines one's income. In both models an income tax has two effects: (a) an effect on the incentive to work (in the labour model) or on the incentive to obtain education (in the other model) and, consequently, on the national income; and (b) an effect on the distribution of after-tax income. Therefore, we often have a conflict between "the size of the pie and its distribution". In this paper we intend to treat explicitly these two effects. By doing so I think that we will obtain more insight into the problems of income taxation and we will be able to give simple diagrammatic proofs of the properties of the optimal tax (see Mirrlees [2], Phelps [3] and Sheshinski [6]). Only the labour model is analysed here in detail. The results, however, can be easily extended to the education model as well (see [4], ch. 3). We will discuss both the additive and the max-min criterion. The assumptions employed are essentially those used elsewhere in the literature except that we do not require Phelps' unrealistic assumption that the leastadvantaged member of society is incapable of producing any income.

1. THE LABOUR MODEL

This model is due to Mirrlees [2]. For convenience we shall recall its main characteristics. There are only two commodities—consumption (denoted by x) and labour services (denoted by y)— and a continuum of consumers. Each consumer is identified by his skill, n, where the range of values of n is some closed and bounded interval $[N_1, N_2]$, with $N_1 \ge 0$. We denote by F(n) the number of persons with skills n or less. F is assumed to be continuous and strictly increasing on $[N_1, N_2]$. All individuals have the same tastes over (x, y), represented by u(x, y), and have the same endowment of leisure—A. It is assumed that u(x, y) is strictly quasi-concave, twice continuously differentiable, strictly increasing in x, strictly decreasing in y and that both goods have diminishing marginal utilities. We also assume that both consumption and leisure are normal goods. I have elsewhere shown (see [5]) that for the additive social welfare function to make sense we must assume that $u_{xy} > 0$, though, for our purposes, it is sufficient to assume $u_{xy} \ge 0.^1$ A person with skill n who works y hours earns a gross income of $z = ny.^2$ His consumption is equal to his net income which is z - T(z), where T is the tax function. For each T, we denote by $[x_T(n), y_T(n)]$ the utility-maximizing bundle chosen by person $n.^3$ The functions $x_T(n), y_T(n)$, and $z_T(n) = ny_T(n)$ are called, respectively, the consumption, the labour supply, and gross

income functions (under T). It is assumed that $-u_y/u_x \to \infty$ as $y \to A$, so that $y_T(n) < A$ for all T and n. We define $u_T(n)$ as the maximum utility level achieved by person n when the tax is T, namely: $u_T(n) = u[x_T(n), y_T(n)]$.

As our social ordering we consider alternatively two criteria: an additive social welfare function and a generalized max-min criterion. The additive social welfare function W, is defined by $W[x(\cdot), y(\cdot)] = \int u[x(n), y(n)] dF(n)$, where the vector function $[x(\cdot), y(\cdot)]$ is an allocation in which person n enjoys the bundle [x(n), y(n)]. The latter integral and all other integrals in this paper are taken over $[N_1, N_2]$. The generalized max-min criterion will be defined over only those allocations which can be induced by some income tax, i.e. allocations of the form $[x_T(\cdot), y_T(\cdot)]$ for some T. For these allocations it will be shown in the next section that the utility level enjoyed by any person is an increasing function of his skill, namely: $u_T(n) = u[x_T(n), y_T(n)]$ is an increasing function of n for each T. We then define a generalized max-min criterion as follows: $[x_{T_1}(\cdot), y_{T_1}(\cdot)]$ is socially preferred to $[x_{T_2}(\cdot), y_{T_2}(\cdot)]$ if and only if there exists $n_0 \ge N_1$ such that $u_{T_1}(n) \ge u_{T_2}(n)$ holds on $[N_1, n_0]$ and strict inequality holds on a subset of $[N_1, n_0]$ with a positive measure (with respect to F); two states are socially indifferent to each other if neither of them is socially preferred to the other.⁴ Notice that the increasing monotonicity of u_T plays a crucial role in the latter definition, since it enables us to locate easily the least-advantaged members of society.

The government aim is to choose T so as to maximize social welfare subject to its own budget constraint: $\int T[z_T(n)]dF(n) \ge B$, where B is some pre-determined level of public consumption. The tax function is further restricted to be continuous.

2. PRELIMINARY RESULTS

In this section we will state (without proof) a few technical results. Some of these are proved in [2]; the others in [4].

In the x-y plane all persons have the same map of indifference curves but, because of skill differences, they face different budget lines. It is, however, rather convenient to work in the x-z plane, where all face the same budget line (because they face the same tax



schedule); but since different individuals can achieve the same z by supplying different y's, it follows that people do not have the same preferences over (x, z). The preferences of person n over (x, z) are described by $u^n(x, z) = u(x, z/n)$ for $z \le nA^5$ Person n maximizes $u^n(x, z)$ subject to the constraint that x = z - T(z); the solution is, of course, $[x_T(n), z_T(n)]$ which were defined earlier. The first lemma follows from the normality of consumption and it states that at any point (x_0, z_0) , the slope of the indifference curve of person n_1 is greater than that of person n_2 , whenever $n_1 < n_2$ (see Figure 1).

Lemma 1. Suppose $0 < n_1 < n_2$ and $z_0 \le n_1 A$. Then $-u_z^{n_1}/u_x^{n_1} > -u_z^{n_2}/u_x^{n_2}$ at (x_0, z_0) . Trivial consequences of Lemma 1 are the following two corollaries.

Corollary 1. Let $n_1 < n_2$ and $z_0 < z_1 \le n_1 A$. If $u^{n_1}(x_0, z_0) \le u^{n_1}(x_1, z_1)$, then: $u^{n_2}(x_0, z_0) < u^{n_2}(x_1, z_1)$.

Corollary 2. For each T, $z_T(n)$ is non-decreasing in n (see Figure 1, where

 $n_1 < n_2, z_0 = z_T(n_1)$ and $z_2 = z_T(n_2)$).

A person with a higher skill will enjoy a higher utility as long as he makes use of his skill, i.e. as long as he works:

Lemma 2. For each T, $u_T(n)$ is non-decreasing in n everywhere and strictly increasing on the interval $[n/z_T(n)>0]$.

Since ny is continuous in n, it is straightforward to prove the next lemma.

Lemma 3. For each T, $u_T(n)$ is continuous in n.

Finally, it can be shown (see [4]) that with no loss of generality we may assume that:

For each T, the set $\{z/z = z_T(n) \text{ for some } n\}$ is a closed and bounded interval. ...(1)

Notice that (1) implies that net income, z - T(z), is non-decreasing in z (the marginal tax is not higher than 100 per cent anywhere).

3. PROPERTIES OF THE OPTIMAL TAX

We are now in a position to show how a tax with a negative marginal rate somewhere can be improved and thus cannot be optimal. We will start with the additive criterion:

Theorem 1 (Mirrlees). If T is optimal under the additive social welfare function, then it is non-decreasing (i.e. the marginal rate is non-negative).

A Sketch of a Proof. Here, for a better presentation of the argument, it is most convenient (though not necessary) to assume that the optimal tax is linear. Suppose, contrary to the assertion of the theorem, that T is decreasing. Let ABC in Figure 2 be the graph of z-T(z); the slope of ABC being greater than 1. Suppose first that u is of the form u(x, y) = S(x) + R(y) where S and R are concave. We will construct another tax, T_1 , which is feasible and socially preferred to T. Under T, each person n supplies labour services at the level of $y_T(n)$ and his gross income is $z_T(n)$. Suppose temporarily that we forbid people to change their labour supplies and, consequently, their gross incomes in response to the new tax that we are now going to construct. Consider now the line EBD which has a slope of unity; it is the graph of $z-T_1(z)$ for some tax T_1 which has a zero marginal rate everywhere. It is clear from Figure 2 that the tax T_1 increases the burden put on the rich and relieves it from the poor. By lowering or raising EBD, as needed, it can be ensured that at this stage (when gross incomes have not yet been changed), T_1 collects the same amount of revenues as T and is therefore feasible:

$$\int T_1[z_T(n)]dF(n) = \int T[z_T(n)]dF(n) \ge B.$$
 ...(2)

Thus, whatever is taken from the rich is rendered back to the poor. If we denote the consumption of person *n* at this stage by $x_1(n)$, then, by definition, $x_1(n) = z_T(n) - T_1[z_T(n)]$ and we conclude from (2) that: $\int x_1(n)dF(n) = \int x_T(n)dF(n)$. Thus, $x_1(\cdot)$ and $x_T(\cdot)$ have the same mean and it therefore follows from the construction of T_1 that $x_1(\cdot)$ could have been obtained from $x_T(\cdot)$ by a mean-preserving concentration (namely, by an appropriate shift of consumption from the rich to the poor).⁶ Hence, the allocation which prevails in



FIGURE 2

this stage, namely $[x_1(\cdot), y_T(\cdot)]$ is socially preferred to the old one, $[x_T(\cdot), y_T(\cdot)]$; for when u is additive, we have

$$W[x_1(.), y_T(.)] - W[x_T(.), y_T(.)] = \int S[x_1(n)] dF(n) - \int S[x_T(n)] dF(n) > 0,$$

because S is concave and $x_1(\cdot)$ could have been obtained from $x_T(\cdot)$ by a mean-preserving concentration. We now allow people to respond to the new tax T_1 . Each person n then moves from $[x_1(n), y_T(n)]$ to $[x_{T_1}(n), y_{T_1}(n)]$. Since people maximize their utilities, we have $u[x_{T_1}(n), y_{T_1}(n)] \ge u[x_1(n), y_T(n)]$ for all n. Since our social welfare function respects individuals' preferences, we conclude that $[x_{T_1}(\cdot), y_{T_1}(\cdot)]$ is socially preferred to $[x_1(\cdot), y_T(\cdot)]$ which is, in turn, preferred to $[x_T(\cdot), y_T(\cdot)]$. Thus, T_1 is preferred to T. It remains to show that T_1 remains feasible after individuals are allowed to change their labour supplies and, consequently, their gross incomes. Since T_1 has a zero marginal rate everywhere, it follows that these changes in gross incomes do not result in any change in tax payments, so that indeed T_1 remains feasible.

When u is not of the form u(x, y) = S(x) + R(y), we can still employ the assumptions that leisure is a normal good and $u_{xy} \ge 0$ to show that, under any tax, rich people have a lower marginal utility of x than poor people. Therefore, some mean-preserving concentration in the distribution of consumption is still desirable. With this in mind, we can, as before, construct a new tax function which is feasible and preferred to T. For a rigorous proof along these lines, the reader is referred to [4]. QED

It should be clear from Figure 2 that all individuals with incomes below \bar{z} are made better off under T_1 . This establishes, at once, a similar theorem for the max-min case:

Theorem 2. If T is optimal under the max-min criterion, then it is non-decreasing. (This theorem was proved by Phelps under the assumption that $N_1 = 0$, which means that the lowest skilled individual is incapable of producing any income, no matter how much he works.)

It is possible to prove an even stronger theorem for the max-min criterion, but for this purpose we need Lemma 4 below which we now explain. Suppose that T is optimal under the max-min criterion. Person N_1 then has income of $z_T(N_1)$ and pays a tax of $T[z_T(N_1)]$.

No other tax which is equal to T at $z_T(N_1)$ can generate more revenues to the government than T; for otherwise, these extra revenues can be used in order to lower the tax burden imposed on every individual with a skill in a neighbourhood of N_1 , in contradiction to the optimality of T. This proves the following lemma.

Lemma 4. If T is optimal under the max-min criterion, then

$$\int T[z_T(n)]dF(n) \ge \int T_1[z_{T_1}(n)]dF(n)$$

for all T_1 with $T_1[z_T(N_1)] = T[z_T(N_1)].^7$

Theorem 3. If T is optimal under the max-min criterion, then it is strictly increasing. (This result too was proved by Phelps but, again, only for the case $N_1 = 0$.)

Proof. In view of Theorem 2, we need only to show that T is not constant on any interval. Suppose, to the contrary, that T is constant on some interval $[z_1, z_2]$. Let **ABCDEHK** in Figure 3 be the graph of z-T(z); the slope of **BCD** being 1. Choose some



 $z_3 \in (z_1, z_2)$ and let n_1 , n_3 and n_2 be, respectively, the skills of persons for whom the points *B*, *C* and *D* are utility-maximizing bundles under *T*. By (1), such n_1 , n_3 and n_2 do exist. Clearly, $n_1 < n_3 < n_2$. Let *QBREJ* be the indifference curve of person n_3 which passes through *B*. It follows from the normality assumption that the slope of the indifference curve *QBREJ* is equal to 1 at some point, say *R*, to the south-east of *C*. At point *R* draw a line *PRHS* which is tangent to the indifference curve *QBREJ* and has therefore a unity slope. Let n_4 be the skill of a person for whom point *H* is a utility-maximizing bundle under *T*. Clearly, $n_2 \leq n_4$. Now define a new tax T_1 such that the graph of $z - T_1(z)$ is *ABRHK*. Points *B* and *H* remain utility-maximizing bundles under T_1 for persons n_1 and n_4 , respectively. Therefore, all persons with skills in (n_1, n_4) will have their equilibrium positions under T_1 along the curve *BRH*, by Corollary 2. Since, by construction, person n_3 considers

point B to be at least as good as any point along BRH, it follows from Corollary 1 that all persons with skills in (n_1, n_3) prefer point B over all other points along BRH and therefore they will choose point B under T_1 . Since the marginal tax rate is zero along BC, then their tax payments do not change. But all persons with skills in (n_3, n_4) prefer point R over B, by Corollary 1. Therefore, they will have their equilibrium positions under T_1 along the line segment RH, where their tax payments do not fall, as compared to T. Moreover, all persons with skills in the subset (n_3, n_2) in fact increase their tax payments.⁸ Also, it is clear from the construction of T_1 that no person with a skill $\leq n_1$ or $\geq n_4$ is affected by the change of the tax from T to T_1 . Therefore, T_1 generates more revenues to the government than T, in contradiction to Lemma 4.⁹



FIGURE 4

An interesting question concerning the optimal tax is whether it is progressive or regressive. Numerical calculations carried out by Mirrlees showed that the marginal rate is slightly decreasing at high income levels (i.e. a regressive tax). Theorem 4 below reinforces this result for the case of bounded gross incomes (i.e. $z_T(N_2) < \infty$).

Theorem 4. Let T be optimal under any one of our social ordering relations and suppose it has a left-derivative everywhere (denote it by $D^{-}T$). Then $D^{-}T[z_{T}(N_{2})] = 0$. In other words, the marginal rate applicable to the richest person must be zero.¹⁰

Proof. Suppose, contrary to the assertion of the theorem, that $D^{-}T[z_T(N_2)] > 0$ (a negative marginal rate is excluded by theorems 1 and 2). Let *ABC* in Figure 4 be the graph of c(z) = z - T(z) and let *HBE* be an indifference curve of person N_2 . Since $D^{-}c[z_T(N_2)] < 1$, it follows from utility-maximization that the slope of this indifference curve is less than 1 at point *B*. Now define a new tax T_1 as follows:

$$T_1(z) = T(z)$$
 for $z \le z_T(N_2)$ and $T_1(z) = T[z_T(N_2)]$ for $z \ge z_T(N_2)$.

The graph of $c_1(z) = z - T_1(z)$ is then *ABD*, the slope of *BD* being unity. Obviously, no one is worse-off under T_1 . Moreover, it is clear from Figure 4 that person N_2 ends up better-off under T_1 by moving to some point along *BD*. (He can do so because, by assumption, $y_T(N_2) < A$.) Then, by continuity, all persons with skills in some neighbourhood of N_2 are

better-off under T_1 . Therefore, T_1 is socially preferred to T according to any individualistic social ordering criterion. That T_1 is feasible is seen by observing that any point along BD results in at least as much tax payment as any point along AB. QED

The reader may have noticed that the preceding theorem excludes the possibility of having the marginal rate of the optimal tax increasing at a neighbourhood of $z_T(N_2)$. For in such a case it will have to be negative at income levels below $z_T(N_2)$. Thus, the optimal tax cannot be progressive everywhere.

First version received November 1973; final version accepted January 1975 (Eds.).

This paper is based on a chapter of the author's doctoral dissertation submitted to the Department of Economics at MIT in May 1974. The author wishes to thank the members of his thesis committee, Peter A. Diamond and Robert M. Solow, for many helpful comments and suggestions. Errors and opinions are, of course, his own.

NOTES

1. These assumptions are slightly stronger than Mirrlees' ones, but I do not find them particularly restrictive.

2. The more general case of z = H(n, y) can also be dealt with, provided some suitable assumptions on the function H are made.

3. This bundle is not necessarily unique, but it can be shown that the set of n's for which this is the case is, at most, countable and may thus be ignored.

4. In the case of a discrete number of individuals, this ordering is equivalent to the following rule: when comparing two social states choose that state for which the minimum utility enjoyed by anyone is higher; in case of a tie, go to the next-to-min and so forth....

5. For person n = 0, the quotient 0/0 is taken here to be 0.

6. We define a mean-preserving concentration as the inverse of a mean-preserving spread. For a definition of a mean-preserving spread the reader is referred to Atkinson [1].

7. A special case of this lemma is essentially used by Phelps to derive his results.

8. Notice that, by assumption $F(n_2) - F(n_3) > 0$.

9. In Figure 3 we have drawn the graph of z - T(z) to intersect line *PRHS*. If this graph does not do so, we define $z - T_1(z)$ to have *ABRHS* as its graph and we can again prove that T_1 generates more revenues than T.

10. In fact, the proof which follows establishes a more general statement: consider any social ordering criterion which is individualistic and let T be optimal under this criterion. If T is non-decreasing, then $D^{-}T[z_{T}(N_{2})] = 0$.

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