Optimal Direct (non linear) Taxation

A model written by

James Mirrlees

Review of Economic Studies, 1971

A story told by

Amedeo Spadaro

Ensae 2009

Basic question:

- it is possible to solve optimally the equity efficiency trade off?
- There is an explanation/justification of observed non linear tax schedules

Outline of the talk:

-Introduction: Franco-Anglo contributions....
-The Mirrlees model:

- Structure
- Components
- Novelty
- Importance
- Resolution techniques
- Economic interpretation of the results

Art 13 of **Déclaration des Droits de l'homme et du citoyen du 26 août 1789**

Pour l'entretien de la force publique, et pour les dépenses d'administration, une contribution commune est indispensable: elle doit être également répartie entre tous les citoyens, en raison de leurs facultés.

Edgeworth (1897)

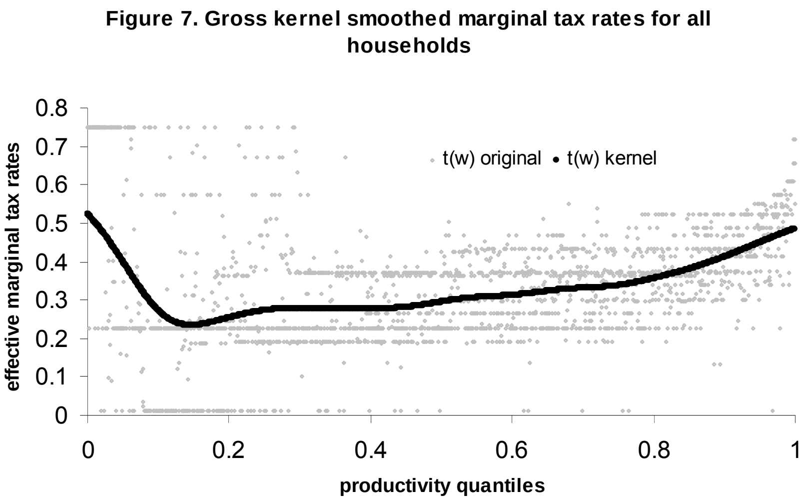
Equal Absolute Sacrifice	$dUo=dU1$
Equal Proportional Sacrifice	$dUo/Uo=dU1/U1$
Equal Marginal Sacrifice	$G'(U)U'()=\lambda$

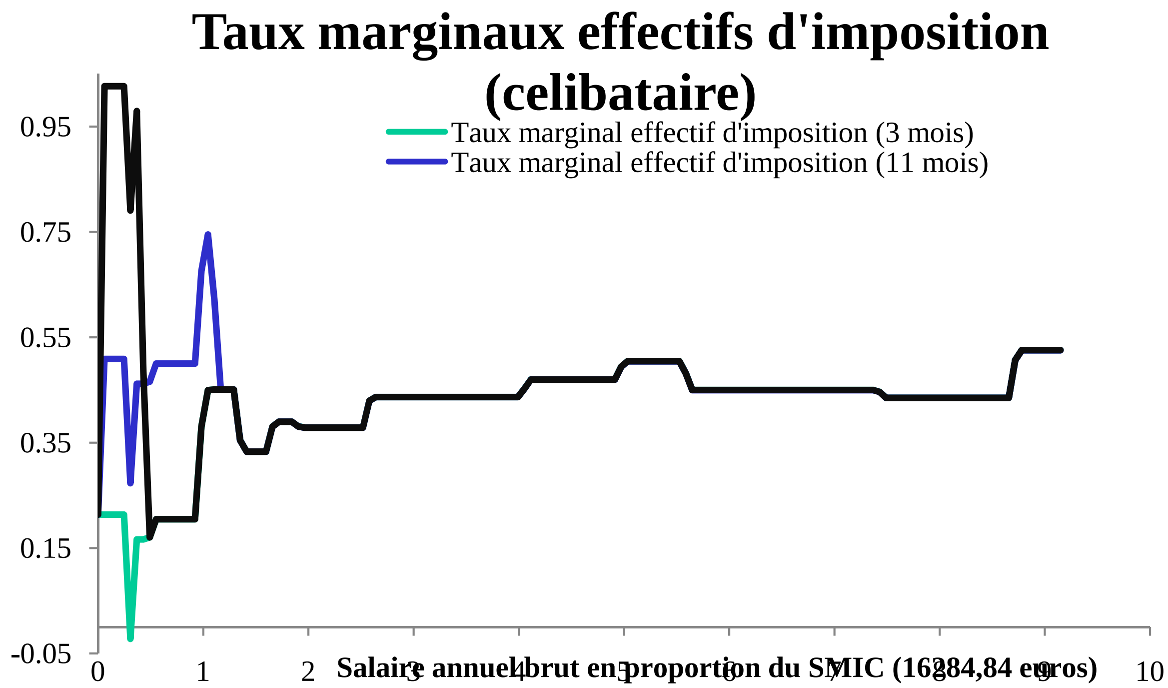
Sidgwick (1883)

Problem of (dis)incentives:

1. The size of the cake
2. Population increases (maltusian arguments)
3. If fully equality is imposed then diversity, progress and liberty are eliminated

First attempts: linear taxation as a replication of Ramsey indirect taxation model. But in reality………





The structure of the Mirrlees model:

- A Social Welfare Function
- A distribution of the productivities
- The Agent behavior (single crossing hp, elasticities)
- The production set (perfect competition and constant returns to scale)
- The informational frictions (an incentive compatibility story...)

The theory

$$V[w,T(\,)] = U(c^*,L^*); \quad (c^*,L^*) = \arg\max[U(c,L); \mathrm{c} = wL - T(\,)]$$

$$V[w,w] = U(c(w),Y(w)/w)$$

$$V[w,w'] = U(c(w'),Y(w')/w)$$

$$\boxed{\begin{gathered} \underset{T()}{Max} \int_{w_0}^{Z} G\{V[w,T(\,)]\} f(w)dw \\ \int_{w_0}^{Z} T(wL^*) f(w)dw \geq Q \\ V[w,w] \geq V[w,w'] \end{gathered}}$$

$$U(c,L) = c - B(L) \qquad B(L) = (1+\frac{1}{\varepsilon})^{-1} L^{1+\frac{1}{\varepsilon}} \qquad L^* = w^{\varepsilon}[1 - T'(wL^*)]^{\varepsilon}$$

$$H\left[L(w),V(w),\mu(w),\lambda\right]=\left[G(V)+\lambda\left(wL-c-Q\right)\right]f(w)+\mu(w)\frac{L}{w}B_L(L)$$

$$H\left[L(w),V(w),\mu(w),\lambda\right]=\left[G(V)+\lambda\left(wL-V(w)-B(L)-\bar{Q}\right)\right]f(w)+\mu(w)\frac{L}{w}B_L(L)$$

(p. foc 1) $$\frac{\partial H()}{\partial L}=\lambda\left(w-B_L\right)f(w)+\mu(w)\frac{\left(LB_L\right)_L}{w}=0$$

(p. foc 2) $$\frac{\partial H()}{\partial V}=-\frac{\partial\mu(w)}{\partial w}\Rightarrow\left[G'(.)-\lambda\right]f(x)=-\frac{\partial\mu(w)}{\partial w}$$

that, after integration and making use of the transversality condition $\mu(Z)=0$

implies that $\int_w^Z\left(1-\frac{G'(.)}{\lambda}\right)f(x)dx=-\frac{\mu(w)}{\lambda}$

Consolidating the two and making use of the f.o.c of problem (1.2) we obtain the condition (4) on the marginal tax rate *t(y)*.

$$\frac{t(y)}{1-t(y)}=\left(1+\frac{1}{\varepsilon}\right)\frac{1-F(w)}{wf(w)}\left[1-S(w)\right]$$

$$S(w)=\frac{1}{\left[1-F(w)\right]}\int_w^Z\frac{G'\left[V(x,T(xL)\right]}{\lambda}f(x)dx$$

To understand the social planner arbitrage:

Imagine a small increase of the tax paid by W agents: dT

-TAX REDUCTION: $$TR = \frac{t(W)}{1-t(W)} \cdot \frac{W.f(W)}{1+1/\varepsilon}.dT$$

-TAX INCREASE: $$TI = [1-F(W)].dT$$

-LUMP SUM REDISTRIBUTION: $TI - TR$

-IN TERMS OF WELFARE: A + B + C =0

$$0 + [1-F(W)].S(W).dT + (TI - TR).S(w_0) = 0$$

The main qualitative results:

A) t(y) always lies between 0 and 1
from f.o.c. of agent utility maximization

B) T(highest productivity)= T(lowest productivity)=0

C) Compute T(y) and see what it looks like

Saez (2002) discrete version model

The Social Planner problem is: $\text{Max}_{[T_0,\ldots T_I]} \int_{w_0}^{Z} \alpha(w)V(w)f(w)dw$

$$\text{s.t}: \quad (c_i^*, i^*) = \text{Argmax}\{U(w, c_i, i);\ c_i = Y_i - T_i,\ i \in [0,1,..I]\}$$

$$V[w] = U(w, c^*, i^*)$$

$$\sum_i h_i T_i \geq \overline{T}$$

h_i is the % of agents choosing *I*; T_i is net tax paid by group *I*

The optimal tax formula is:

$$\frac{T_i - T_{i-1}}{C_i - C_{i-1}} = \frac{1}{\mu_i h_i} \sum_{j \geq i}^{I} h_j \left[1 - g_j - \chi_j \frac{T_j - T_0}{C_j - C_0}\right]$$

-C_i is the net household income of group *i*,

-Non-workers receive benefits – T_o, by definition identical to C_o.

$$g_i = \frac{1}{\lambda h_i} \int_{w \in i} \alpha(w) \frac{\partial V(w)}{\partial c_i} f(w)$$

is the marginal weight the government assigns to group *i*. This weight represents the value (expressed in terms of public funds) of giving an additional dollar to an individual in group *i*. λ is the Lagrange multiplier associated to the aggregate budget constraint.

The **intensive elasticity** is defined as:

$$\mu_i = \frac{C_i - C_{i-1}}{h_i} \frac{dh_i}{d(C_i - C_{i-1})}$$

And it is related with the classical one with:

$$\mu_i = \frac{Y_i}{Y_i - Y_{i-1}} \varepsilon_i$$

The **extensive elasticity** is

$$\chi_i = \frac{C_i - C_0}{h_i} \frac{dh_i}{d(C_i - C_0)}$$

Application of the Mirrlees model to some European Country
España, France, UK, Italia (10000 observations); $\varepsilon = 0.1$ et $\varepsilon = 0.5$.

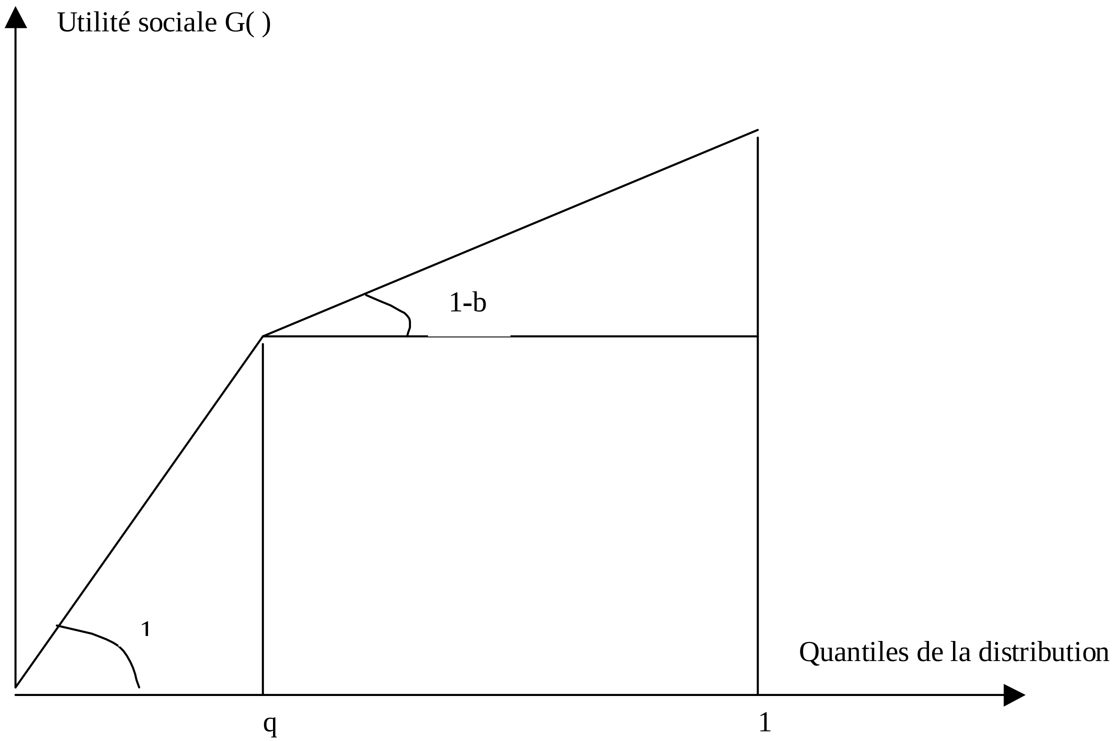
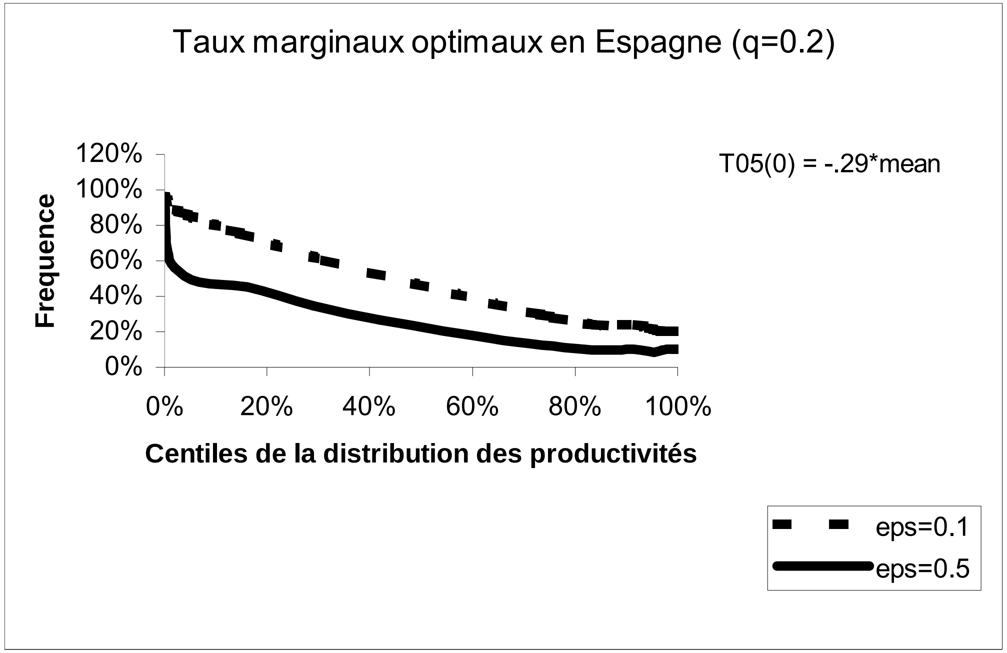


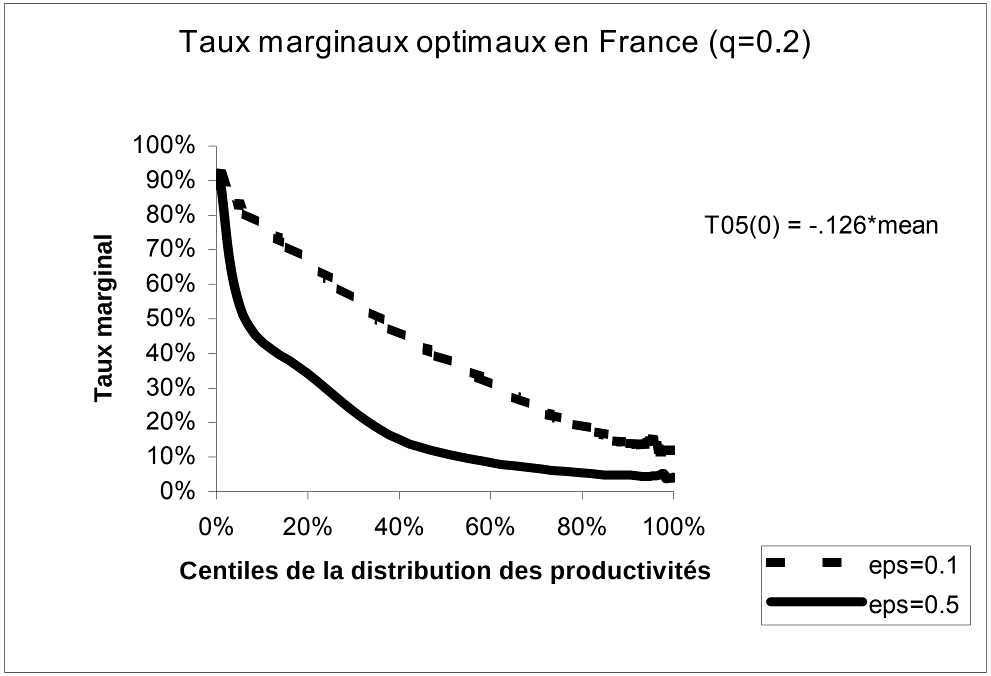
Figure 1. Forme de la fonction de bien-être social [G( )]

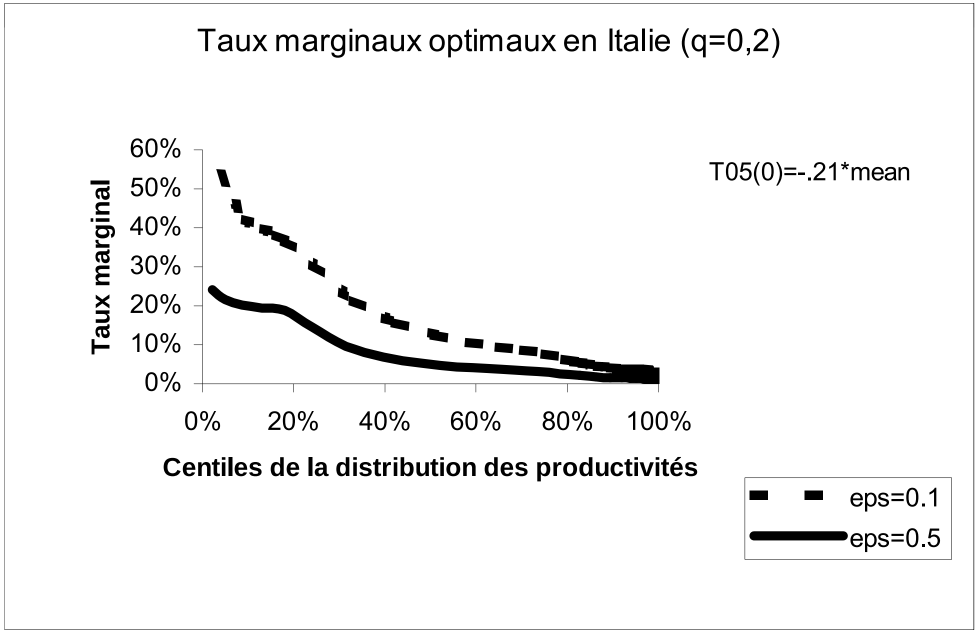
$$(1-q)H_0 + q\frac{H_0}{b} = 1$$

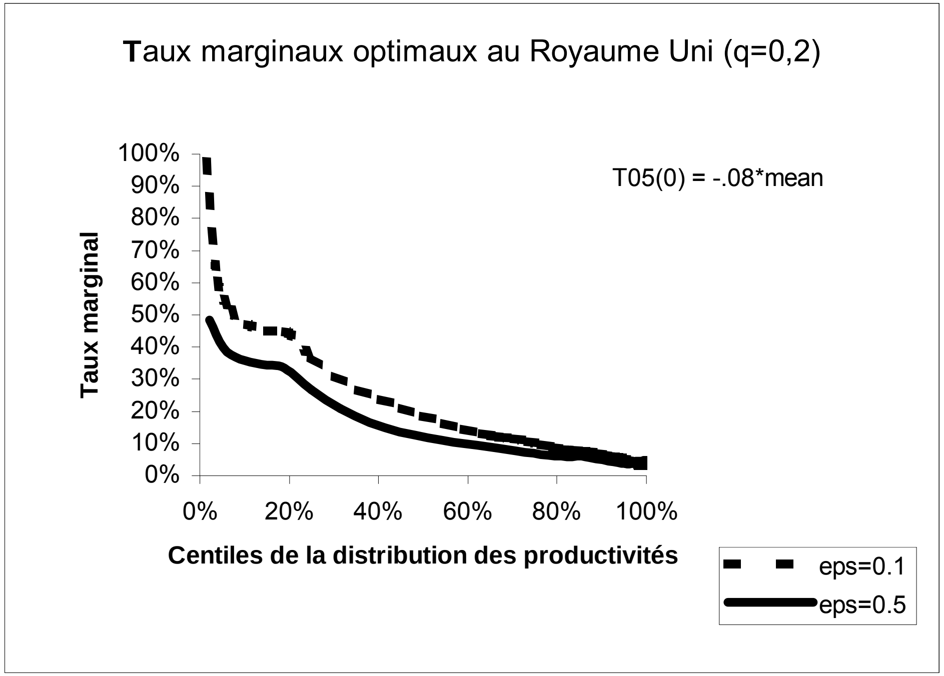
- q = 20%
- *b* calibrated in order to guarantee a minimum income equal to 50% of average income in each country (with $\varepsilon = 0.1$) [T(0)].

$$\begin{cases} \dfrac{t}{1-t} = \left[1+\dfrac{1}{\varepsilon}\right]*\left[\dfrac{1}{wf(w)}\right]*\left[1-F(w)\right]*\left[1-H_0\right] & \text{if } w > w_q \\[2ex] \dfrac{t}{1-t} = \left[1+\dfrac{1}{\varepsilon}\right]*\left[\dfrac{1}{wf(w)}\right]*\left[F(w)*\left(\dfrac{H_0}{b}-1\right)\right] & \text{if } w \le w_q \end{cases}$$


Taux marginaux optimaux en Espagne (q=0.2)
Frequence
120%
100%
80%
60%
40%
20%
0%
T05(0) = -.29*mean
0%
20%
40%
60%
80%
100%
Centiles de la distribution des productivités
eps=0.1
eps=0.5


Taux marginaux optimaux en France (q=0.2)
Taux marginal
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
T05(0) = -.126*mean
0%
20%
40%
60%
80%
100%
Centiles de la distribution des productivités
eps=0.1
eps=0.5


Taux marginaux optimaux en Italie (q=0,2)
Taux marginal
60%
50%
40%
30%
20%
10%
0%
T05(0)=-.21*mean
0%
20%
40%
60%
80%
100%
Centiles de la distribution des productivités
eps=0.1
eps=0.5


Taux marginaux optimaux au Royaume Uni (q=0,2)
Taux marginal
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
T05(0) = -.08*mean
0%
20%
40%
60%
80%
100%
Centiles de la distribution des productivités
eps=0.1
eps=0.5

"Inversion of the optimal problem" (see Kurz 1968, Ahmad and Stern 1984), Bourguignon and Spadaro (2000; 2007).

$$H\left[L(w),V(w),\mu(w),\lambda\right]=\left[G(V)+\lambda\left(wL-c-Q\right)\right]f(w)+\mu(w)\frac{L}{w}B_L(L)$$

$$\frac{\partial H()}{\partial L}=\lambda\left(w-B_L\right)f(w)+\mu(w)\frac{\left(LB_L\right)_L}{w}=0$$

$$\frac{\partial H()}{\partial V}=-\frac{\partial\mu(w)}{\partial w}\Rightarrow\left[G'(.)-\lambda\right]f(x)=-\frac{\partial\mu(w)}{\partial w}$$

$$\frac{t(y)}{1-t(y)}=\left(1+\frac{1}{\varepsilon}\right)\frac{1-F(w)}{wf(w)}\left[1-\frac{S(w)}{S(w_0)}\right]$$

$$S(w)=\frac{1}{\left[1-F(w)\right]}\int_w^Z\frac{G'\left[V(x,T(xL)\right]}{\lambda}f(x)dx$$

It is well-known (for the **Mangasarian theorem**) that the Pontryagin Maximum Principle that leads to the optimality conditions (p. foc 1) and (p.foc 2) are **necessary and sufficient** provided that *H(.)* is ***differentiable and concave*** in the variables *(L,V)* jointly. Given that in our case *H* is separable in *(L,V)*, the Mangasarian theorem needs that:

D) $\dfrac{\partial^2 G(.)}{\partial V^2} < 0$ (e.g. the concavity of social welfare function. It ensures the concavity of the Hamiltonian with respect to *V*).

E) $\dfrac{\mu(w)}{\lambda} < wf(w)\dfrac{B_{LL}}{[LB_L]_{LL}}$ (from $\dfrac{\partial^2 H()}{\partial L^2} < 0$ it ensures the concavity of the Hamiltonian with respect to the control variable *L*).

Consistency with agent maximizing behavior and Spence-Mirrlees condition (this condition ensure that the first order approach to the incentive compatibility constraint is sufficient, see Ebert 1992).

A) $t(y) < 1$ for any *w* (from the f.o.c. of problem 1.2);

B) $t'(y) > \dfrac{U_{LL} + U_{cc}[w(1-t(y))]^2}{w^2 U_c} = \dfrac{U_{LL}}{w^2} = -\dfrac{(1-t(y))}{\varepsilon y}$ for any *w* (from the s.o.c. of problem 1.2);

C) $\dfrac{\partial C}{\partial w} > 0$ without taxes; this is the Spence-Mirrlees condition

If one of the conditions A, B, C, D and E does not hold, then it is the whole optimization concept behind Mirrlees framework that would become doubtful. It would indeed be very difficult to assume that the redistribution authority attempts to maximize a non-concave welfare function if other than trivial redistributions policies are observed.

Of course, from a mathematical point of view we cannot completely rule out a maximizing behavior. The point is that we are not able to characterize it.

If, on the contrary, conditions A, B, C, D and E hold then:

$$S(w) = 1 - \frac{t(y)}{1-t(y)} \frac{\varepsilon}{1+\varepsilon} \frac{w.f(w)}{1-F(w)}$$

$$\frac{t(y)}{1-t(y)} = \left(1 + \frac{1}{\varepsilon}\right) \frac{1-F(w)}{wf(w)} \left[1 - \frac{S(w)}{S(w_0)}\right]$$

$$\frac{G'\left[V(w,T(y)\right]}{\lambda} = 1 + \left(\frac{\varepsilon}{1+\varepsilon}\right)\left(\frac{t(y)}{1-t(y)}\right)\left[1 + \eta(w) + \nu(y)\frac{1+\varepsilon}{1-t(y)+\varepsilon\nu(y)t(y)}\right]$$

$$\nu(y) = yt'(y)/t(y) \qquad \eta(w) = wf'(w)/f(w)$$

Proposition 1. *A necessary condition for the social welfare function making the observed effective marginal tax rate schedule, t(w), optimal with respect to the observed distribution of productivities, f(w) to be Paretian - e.g. non-decreasing everywhere- is that :*

$$t(w) \leq \frac{\frac{1+\varepsilon}{\varepsilon}\frac{1-F(w)}{w.f(w)}}{1+\frac{1+\varepsilon}{\varepsilon}\frac{1-F(w)}{w.f(w)}} \quad \text{for all } w \in [w_0, Z]$$

Alternative interpretation: **Laffer Bound Test**

Where the distribution may be approximated by a Pareto with parameter *a*, given that $\frac{w.f(w)}{[1-F(w)]} = a$, it comes that: $t(w) \leq \frac{1+1/\varepsilon}{1+1/\varepsilon + a}$

For instance, with not unreasonable figures like *a* = 3 and ε =0.5, this condition states that a redistribution system where the marginal tax rate would exceed 50 per cent could be deemed 'optimal' only on the basis of a non-Paretian social welfare function.

Proposition 2*. If the elasticity of the marginal tax rate and the density function are bounded, then there exists a threshold for the wage elasticity of labor supply below which the social welfare function is necessarily non-decreasing everywhere.*

$$\frac{G'\left[V(w,T(y)\right]}{\lambda} = 1 + \left(\frac{\varepsilon}{1+\varepsilon}\right)\left(\frac{t(y)}{1-t(y)}\right)\left[1+\eta(w)+\nu(y)\frac{1+\varepsilon}{1-t(y)+\varepsilon\nu(y)t(y)}\right]$$

This property shows **the importance of the assumption made on the wage sensitivity of labor supply** to judge the optimality of a given redistribution system. Any redistribution system may be said to optimize a Paretian social welfare function, provided that the redistribution authority has a low enough estimate of the wage elasticity of labor supply.

Proposition 3*. Wherever the marginal tax rate is increasing with income, a sufficient condition for the social welfare function to be everywhere non-decreasing is:*

$$t(w) \leq \frac{1+\varepsilon}{1-\eta(w)\varepsilon} \qquad (12)$$

Again, this proposition is directly derived from (9). It is of relevance in connection with the discussion on whether the marginal tax rate curve must be U-shaped – see Diamond (1998) and Saez (2001). In that part where the marginal tax rate is increasing, that is for high incomes, (12) gives an upper limit for the marginal tax rate – in the reasonable case where $\eta(w)$ is negative of course.

Income Effects

$U(c, L) = A(c) - B(L)$

$$\frac{t(y)}{1-t(y)}\psi[c(w)] = \left(1+\frac{1}{\varepsilon}\right).\frac{1-F(w)}{w.f(w)}.\left[\bar{\psi}[c(w)] - S(w)\right]$$

$$\psi[c(w)] = \frac{1}{A'(c)} \qquad \bar{\psi}[c(w)] = \frac{1}{1-F(w)}\int_w^z \psi[c(w)]f(x)dx$$

Proposition 4. *A necessary condition for the social welfare function to be Paretian is that :*

$$t(y) \le \frac{\frac{1+\varepsilon}{\varepsilon}\frac{1-F(w)}{w.f(w)}\frac{\bar{\psi}[c(w)]}{\psi[c(w)]}}{1+\frac{1+\varepsilon}{\varepsilon}\frac{1-F(w)}{w.f(w)}\frac{\bar{\psi}[c(w)]}{\psi[c(w)]}} \quad \text{for all } w \in [w_0, Z]$$

Note that $\frac{\bar{\psi}[c(w)]}{\psi[c(w)]} \ge 1$ implies that the inclusion of income effect mitigate the possibility to be Non Paretian.

Empirical Implementation: A) **individual vs household level**; **B) net vs gross rate of taxation.** 3 key ingredients

1) estimates of the elasticity of labor supply, ε

In the case of France, Bourguignon and Magnac (1991), Piketty (1998), Donni (2000), Bargain (2005), Choné et al. (2003) and Laroque and Salanié (2002). Values between **0.1-0.2 are found for men** and an average of **0.5 is found for married women** - and slightly more **(0.6 to 1) if they have children** (Piketty 1998, Pavot and Spadaro 2008. This second result is mainly driven by participation effects.

which is the right one for households?..second member..!

2) the distribution *f(w)*

you can use wages or "productivities" i.e: $w = Y^{\frac{1}{1+\varepsilon}} [k(1-t(Y))]^{\frac{-\varepsilon}{1+\varepsilon}}$

3) the marginal rate of taxation, *t(w):* computed by microsimulation model (**net and gross**)

$$t(y) = \frac{\Delta Taxes + \Delta Benefits}{\Delta Gross\ Income} = 1 - \frac{\Delta Yd}{\Delta y}$$

-*Important: t(w), f(w) and derivatives computed by Adaptive kernel smoothing techniques*.

Problems with:

1) *Irrational behavior*: ...solved à la Hausman
2) *Scarcity of data at the upper tails of the distribution*: .....only for the last 4-5 centiles

Figure 1. Kernel smoothed marginal tax rates for singles: net and gross scenarios

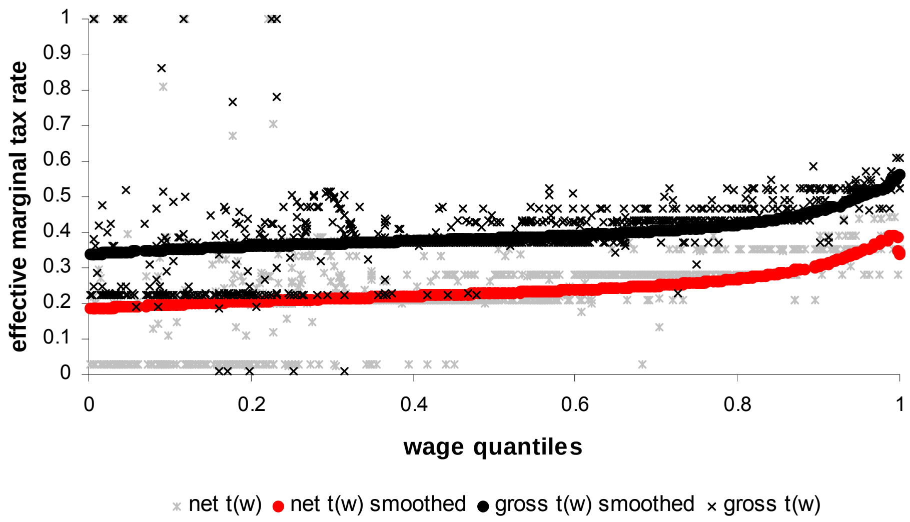


Figure 2. Kernel wage densities for singles: net and gross scenario

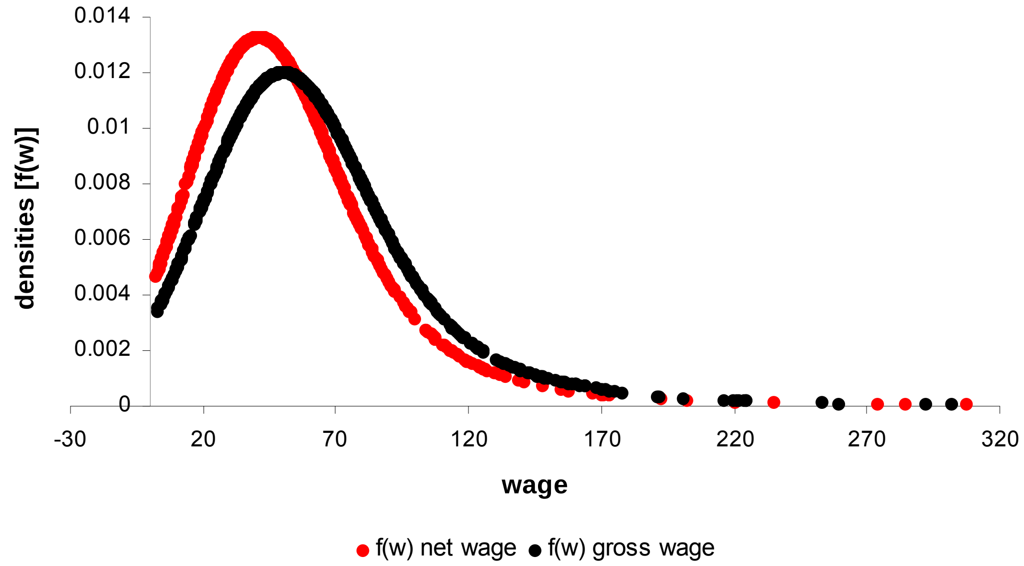


Figure 3. Social marginal welfare for singles (on net wages)

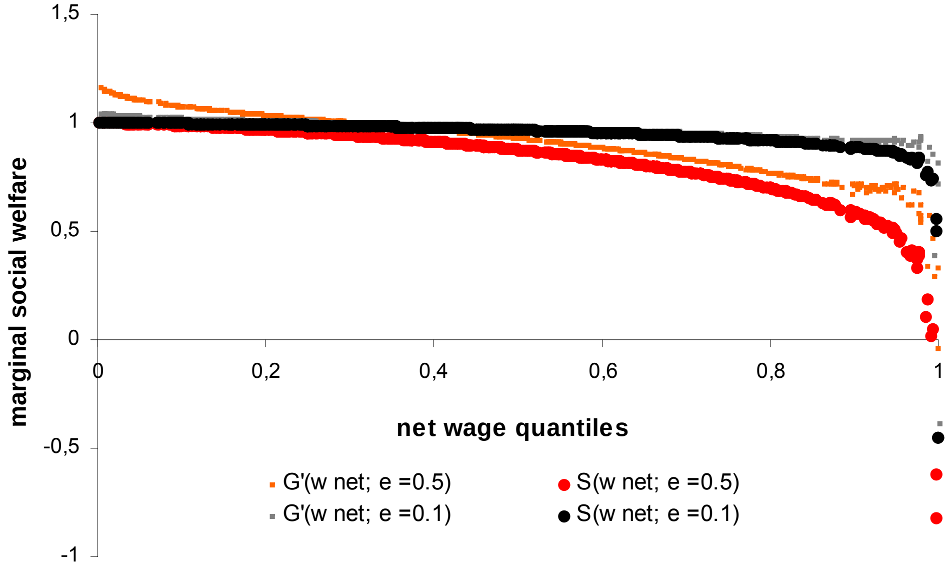


Figure 4. Social marginal welfare for singles (on gross wages)

Figure 5. Kernel productivity densities for singles

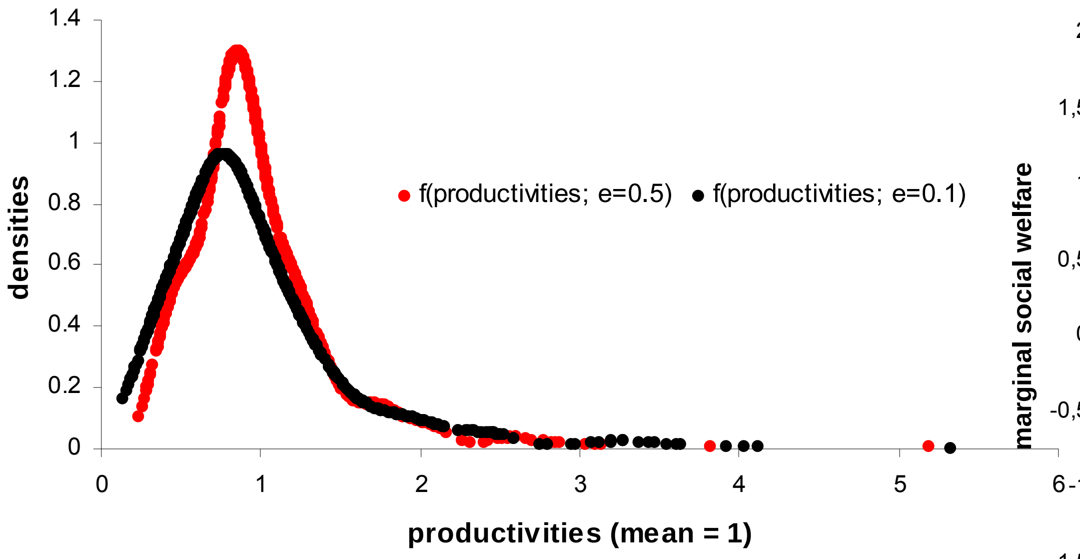


Figure 6. Social marginal welfare for singles (on productivities)

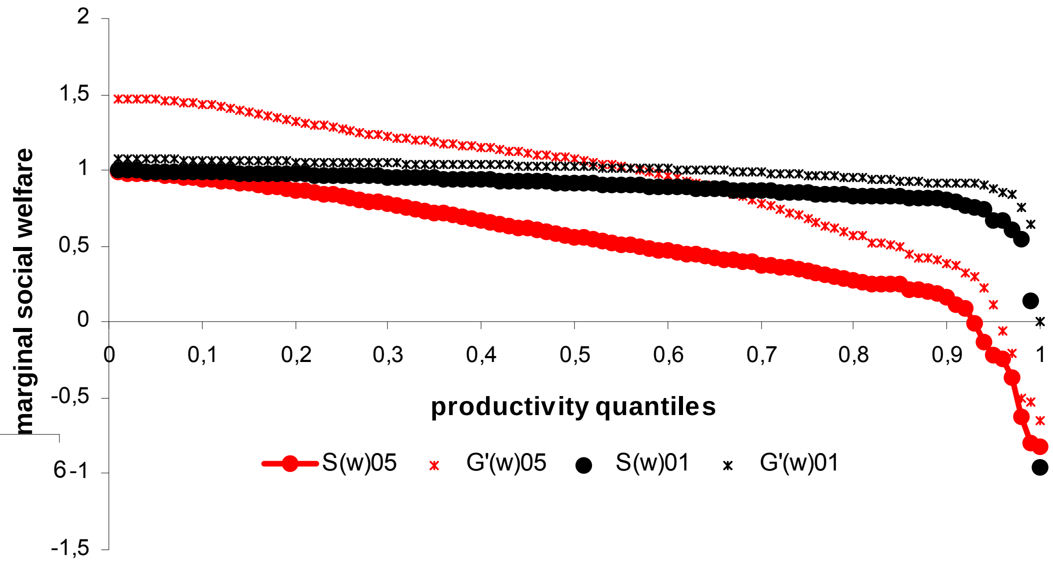


Figure 7. Gross kernel smoothed marginal tax rates for all households

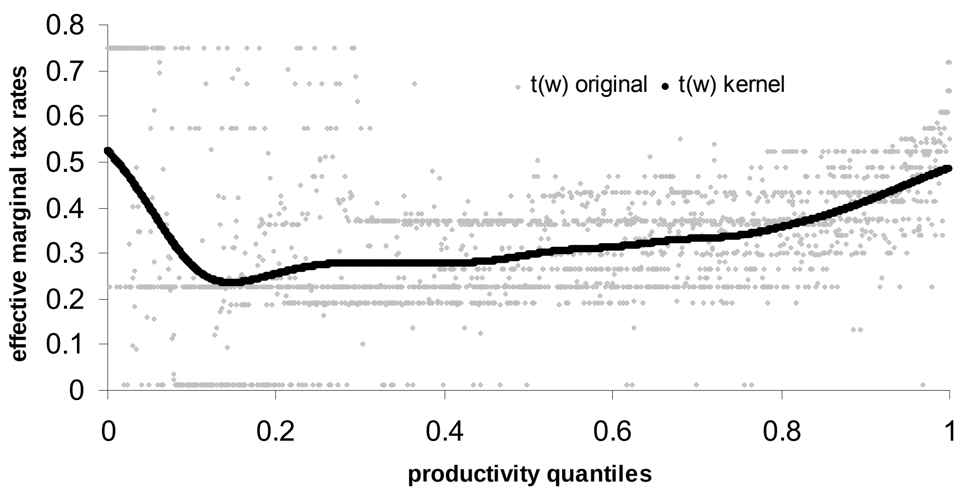


Figure 8. Kernel productivity densities for all households

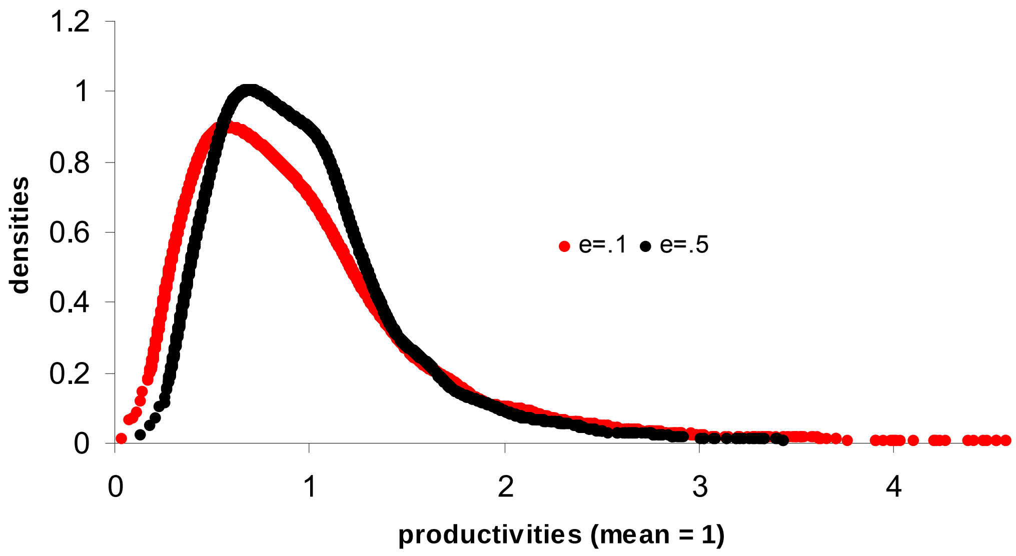


Figure 9. Social marginal welfare for all household (on productivities)

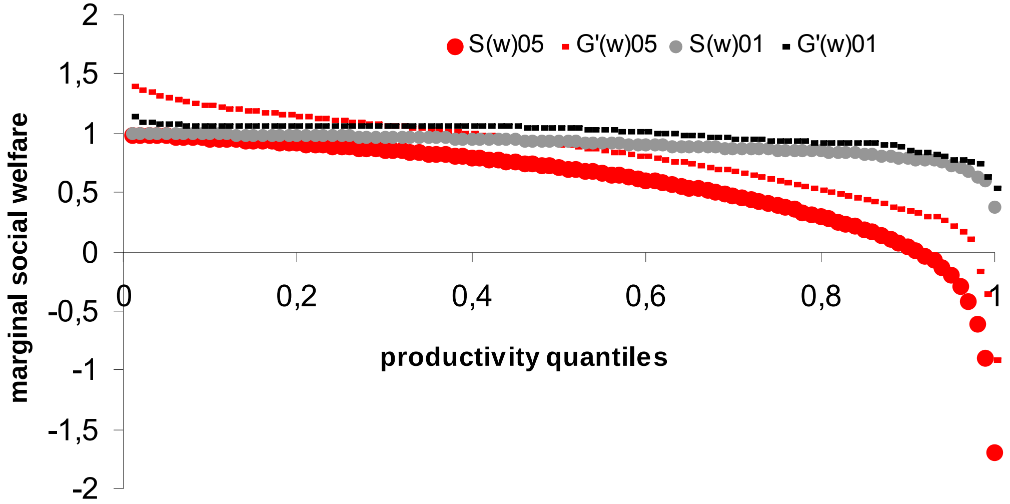
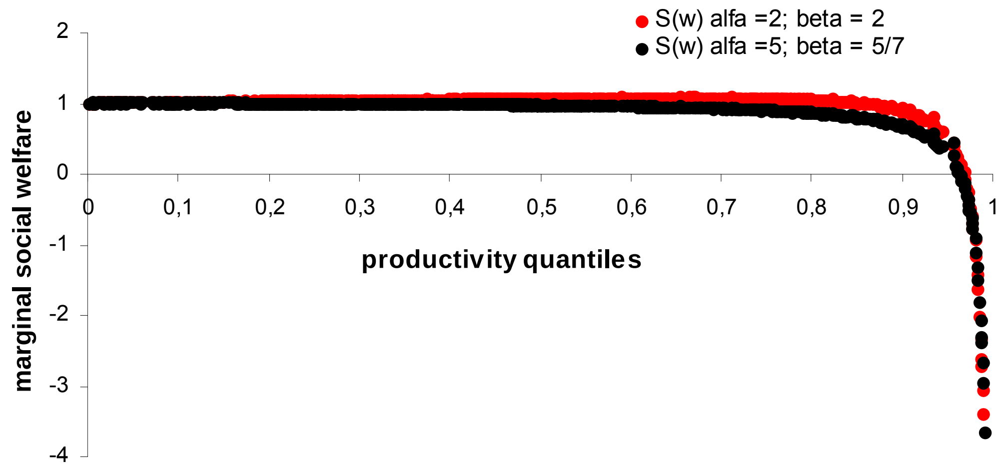


Figure 10. Paretianity test on social marginal welfare for singles (on gross wages) with income effects



Intensive vs Extensive labour supply framework

Saez (2002), [Laroque (2005); Blundell et al. (2006)]

$$\frac{T_i - T_{i-1}}{C_i - C_{i-1}} = \frac{1}{\mu_i h_i} \sum_{j \geq i}^{I} h_j \left[1 - g_j - \chi_j \frac{T_j - T_0}{C_j - C_0} \right]$$

-*Ti* is net tax paid by group *i* and

-*Ci* is the net household income of this group,

-Non-workers receive benefits *-T0*, by definition identical to *C0*.

-*gi*, the marginal weight the government assigns to group *i*. This weight represents the value (expressed in terms of public funds) of giving an additional dollar to an individual in group *i*.

The **intensive elasticity** is defined as:

$$\mu_i = \frac{C_i - C_{i-1}}{h_i} \frac{dh_i}{d(C_i - C_{i-1})}$$

And it is related with the classical one with:

$$\mu_i = \frac{Y_i}{Y_i - Y_{i-1}} \varepsilon_i$$

The **extensive elasticity** is

$$\chi_i = \frac{C_i - C_0}{h_i} \frac{dh_i}{d(C_i - C_0)}$$

$$g_i = 1 - \chi_i \frac{T_i - T_0}{C_i - C_0} - \mu_i \frac{T_i - T_{i-1}}{C_i - C_{i-1}} + \frac{1}{h_i} \sum_{j=i+1}^{I} h_j \left[1 - g_j - \chi_j \frac{T_j - T_0}{C_j - C_0} \right]$$

$$g_0 = (1 - \sum_{i=1}^{I} h_i g_i) / h_0$$

$$g_I = 1 - \chi_I \frac{T_I - T_0}{C_I - C_0} - \mu_I \frac{T_I - T_{I-1}}{C_I - C_{I-1}}$$

Note that if participation elasticity = 0:

$$\frac{T_I - T_{I-1}}{C_I - C_{I-1}} \leq \frac{1}{\mu_I}$$

Equivalent of Proposition 1 (Paretianity condition)

- Sample of singles aged 18 to 65, in which students and individuals with non-labor income above 10 per cent of total income are eliminated

- The final sample used in this exercise contains 1028 singles (963 working).

- The rate of nonlabor force participation (zero yearly earnings reported) for this group is around 9 percent.

- We present only the case in which the redistribution system includes income taxes, assimilated contributions like the 'Cotisation Sociale Généralisée', all non-contributory benefits and the contribution to health insurance (this redistribution system has been referred to as 'gross' in the previous section).

I	Y_i	C_i	T_i	h_i	*F(Y)*
0	0	12000	-12000	0,09	*9%*
1	48857	35919	12939	0,08	*17%*
2	74340	54398	19942	0,09	*26%*
3	91116	64926	26190	0,09	*35%*
4	105954	73144	32811	0,09	*44%*
5	121247	80750	40497	0,09	*53%*
6	135790	87779	48011	0,09	*63%*
7	152870	95747	57122	0,09	*72%*
8	175352	106173	69179	0,09	*81%*
9	215857	123988	91869	0,09	*90%*
10	408454	217915	190539	0,10	*100%*

Scenario				Scenario			
A	i	**Low (0, 1, 2)**	**High (others)**	F	i	**Low (0, 1, 2)**	**High (others)**
	χ	0	0		χ	0.5	0
	ε	0.1	0.1		ε	0.5	0.5
B	i	**Low (0, 1, 2)**	**High (others)**	G	i	**Low (0, 1, 2)**	**High (others)**
	χ	0	0		χ	1	0
	ε	0.5	0.5		ε	0	0.1
C	i	**Low (0, 1, 2)**	**High (others)**	H	i	**Low (0, 1, 2)**	**High (others)**
	χ	0.5	0		χ	1	0
	ε	0	0.1		ε	0.1	0.1
D	I	**Low (0, 1, 2)**	**High (others)**	I	i	**Low (0, 1, 2)**	**High (others)**
	χ	0.5	0		χ	1	0
	ε	0.1	0.1		ε	0.5	0.1
E	I	**Low (0, 1, 2)**	**High (others)**	L	i	**Low (0, 1, 2)**	**High (others)**
	χ	0.5	0		χ	1	0
	ε	0.5	0.1		ε	0.5	0.5

Figure 11. Social marginal weights for singles gross wages (scenario A and scenario B)

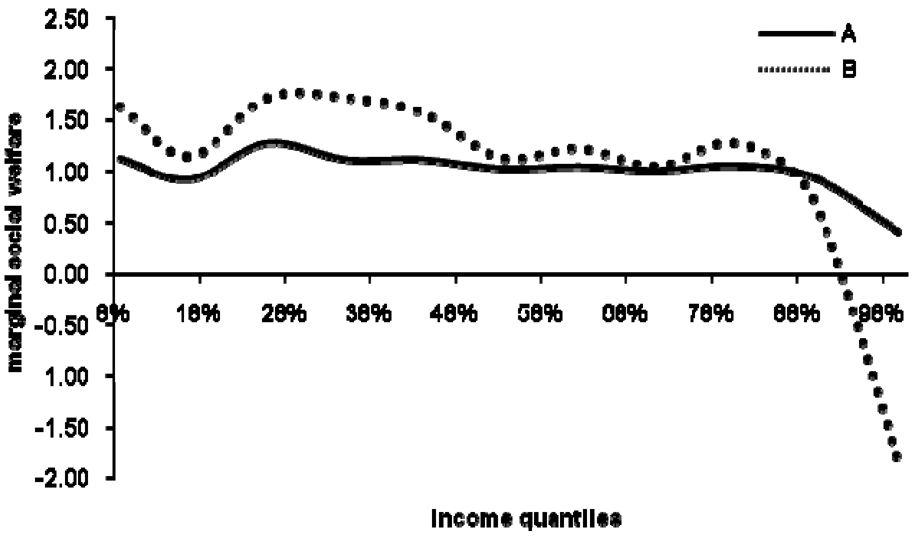


Figure 4. Social marginal welfare for singles (on gross wages)

Optimal Taxation,

Social Contract

and

The Four Worlds of Welfare Capitalism

The starting point:

In their excellent survey about the debate regarding Esping-Andersen’s typology of welfare states, Arts and Gelissen (2002) reconstruct several typologies of welfare states in order to establish, first, whether real welfare states are quite similar to others or whether they are rather unique specimens, and, second, whether there are three ideal-typical worlds of welfare capitalism or more.

They conclude that “***real welfare states are hardly ever pure types and are usually hybrid cases*** *and that the issue of ideal-typical welfare states cannot be satisfactorily answered* ***given the lack of formal theorizing and the still inconclusive outcomes of comparative research****. In spite of this conclusion there is plenty of reason to continue to work on and with the original or modified typologies*”.

Objectives:

a) Check if it is possible to justify the most salient features of existing systems by some optimal tax argument à la “Mirrlees (1971)”. Diamond (1998), Saez (2001, 2002), Salanié (1998), Piketty (1997), Choné and Laroque (2005), Bourguignon and Spadaro (2000; 2002, 2007)

b) Offer a formal theorizing allowing the identification (if possible) of *ideal-typical welfare states* in the spirit of the Esping Andersen (1992) qualitative analysis of European welfare regimes.

What we do:

a) we use the formal setting of the optimal tax theory to try to identify the level of Rawlsianism of some European social planner starting from the observation of the real data and redistribution systems and

b) we use it as a test of the Esping Andersen (and others) classification.

How: with the “Inversion of the optimal problem” technique (see Kurz 1968, Ahmad and Stern 1984), Bourguignon and Spadaro (2007).

Results

a) Redistribution systems in these countries are consistent with the hypothesis of an optimizing redistribution authority.

b) There appears to be a clear coincidence of high decommodification and high Rawlsianism in the Scandinavian, social-democratically influenced welfare states (Denmark). There is an equally clear coincidence of low decommodification and utilitarianism in the Anglo–Saxon liberal model (UK) and in the Southern European welfare states (Italy and Spain). Finally, the Continental European countries (Finland, Germany and France) group closely together in the middle of the scale, as corporatist and etatist. Applied Optimal Taxation validates Esping Andersen.

Definition:

Degree of decommodification:

the degree to which a (social) service is rendered as a matter of right and

the degree to which a person can maintain a livelihood without reliance on the market

Claim:

A redistribution system allowing for a high level of subsidies directed to non working people implies a high level of decommodification

$\Rightarrow$ ***there is a strong analogy between a social planner that want to "decommodificate" individuals and the Rawlsian social planner in an optimal tax model***

Welfare States classification [Arts and Gelissen (2002)]

	Social democracy	Corporativist	Liberal	Southern-European
Degree of decommodification	**Strong**	**Medium**	**Weak**	**Weak**
Ideological reference point	Universalism	Social Hierarchy. Family	Individual responsibility	Family
Representative Countries	Denmark	Finland, Germany, France	UK	Spain, Italy

Theory

The Social Planner problem is: $$\text{Max}_{[T_0,\dots T_I]} \int_{w_0}^{Z} \alpha(w) V(w) f(w) dw$$

$$\text{s.t}: \quad (c_i^*, i^*) = \text{Argmax}\{U(w, c_i, i);\ c_i = Y_i - T_i,\ i \in [0,1,..I]\}$$

$$V[w] = U(w, c^*, i^*)$$

$$\sum_i h_i T_i \geq \overline{T}$$

h_i is the % of agents choosing *I*; T_i is net tax paid by group *I*

The optimal tax formula is: $$\frac{T_i - T_{i-1}}{C_i - C_{i-1}} = \frac{1}{\mu_i h_i} \sum_{j \geq i}^{I} h_j \left[1 - g_j - \chi_j \frac{T_j - T_0}{C_j - C_0}\right]$$

-C_i is the net household income of group *i*,

-Non-workers receive benefits $-T_o$, by definition identical to C_o.

$$g_i = \frac{1}{\lambda h_i} \int_{w \in i} \alpha(w) \frac{\partial V(w)}{\partial c_i} f(w)$$

is the marginal weight the government assigns to group *i*. This weight represents the value (expressed in terms of public funds) of giving an additional dollar to an individual in group *i*. λ is the Lagrange multiplier associated to the aggregate budget constraint.

The **intensive elasticity** is defined as: $\mu_i = \frac{C_i - C_{i-1}}{h_i} \frac{dh_i}{d(C_i - C_{i-1})}$

And it is related with the classical one with: $\mu_i = \frac{Y_i}{Y_i - Y_{i-1}} \varepsilon_i$

The **extensive elasticity** is $\chi_i = \frac{C_i - C_0}{h_i} \frac{dh_i}{d(C_i - C_0)}$

The inversion of the optimal problem (Bourguignon and Spadaro 2000, 2007)

$$g_i = 1 - \chi_i \frac{T_i - T_0}{C_i - C_0} - \mu_i \frac{T_i - T_{i-1}}{C_i - C_{i-1}} + \frac{1}{h_i} \sum_{j=i+1}^{I} h_j \left[1 - g_j - \chi_j \frac{T_j - T_0}{C_j - C_0} \right]$$

$$g_0 = (1 - \sum_{i=1}^{I} h_i g_i) / h_0$$

$$g_I = 1 - \chi_I \frac{T_I - T_0}{C_I - C_0} - \mu_I \frac{T_I - T_{I-1}}{C_I - C_{I-1}}$$

Relevant questions:

1) Are social weights decreasing? (Global aversion to inequality)

2) Weights of group 0 ? (Rawlsianism and Decommodification)

3) What drives the results? (efficiency concerns….)

Data and selection

Country	Data	Year	size of selected	weighted no. of singles	proportion of all singles
Denmark	European Community Household Panel	1995	574	417,945	40%
Finland	Income distribution survey	1998	1193	421,447	38%
France	Household Budget Survey	1994/5	1639	3,615,095	40%
Germany	German Socio-Economic Panel	1998	1387	8,242,791	43%
UK	Family Expenditure Survey	1995/6	1227	5,172,454	47%
Italy	Survey of Households Income and Wealth	1996	1482	3,651,857	51%
Spain	European Community Household Panel	1996	738	1,297,780	37%

Selection criteria:

- singles

- potential workers (no pensioners, no student, working age 18-60)

- no substantial capital income (max 10% of earned income)

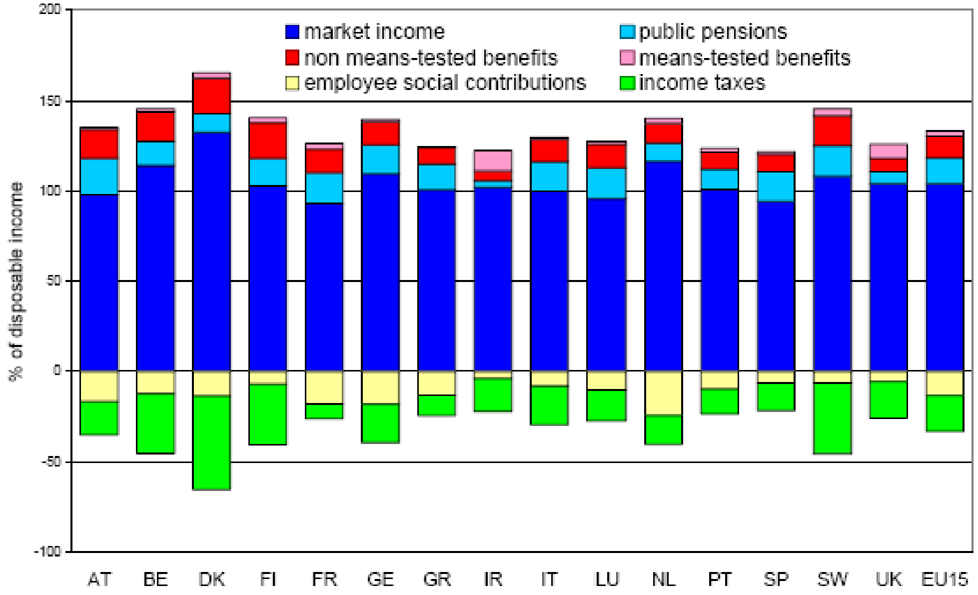
Incomes:

Y_i: income from wage and self-employment income

C_i: includes taxes, contributions, transfers and (contributory) unemployment benefits (treated as redistributive transfer here)

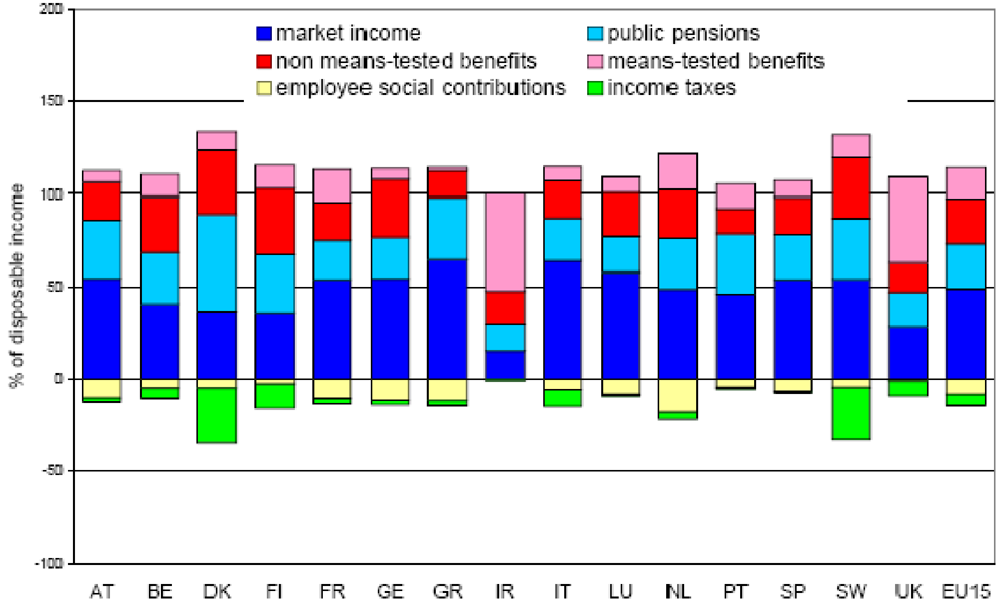
Computed with **EUROMOD**

Market Income, Taxes, Benefits and Replacement Incomes, as a proportion of disposable incomes



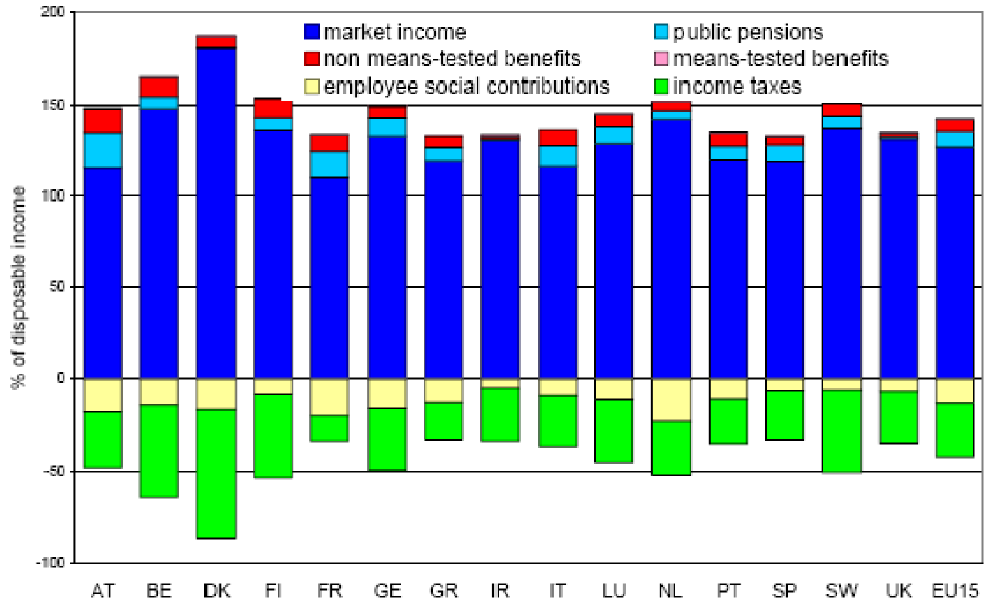
Source: EUROMOD (See http://www.iser.essex.ac.uk/msu/emod/emodstats/index.php)

All households



Source: EUROMOD (See http://www.iser.essex.ac.uk/msu/emod/emodstats/index.php)

Bottom decile



Source: EUROMOD (See http://www.iser.essex.ac.uk/msu/emod/emodstats/index.php)

Top decile

Application: defining groups

Arbitrary definitions but attempt to make it comparable across countries

Type 0: from 0 to part-time paid at minimum wage (rare observation in-between)

Type 1: working poor (up to 1.3x the minimum wage)

Type 2: up to median income

Type 3: up to 1.5 x median income

Type 4: up to 2 x median income

Type 5: above

Cut-off points (monthly gross income in EUR)

groups	Denmark	Finland	France	Germany	Italy	Spain	UK
0	0	0	0	0	0	0	0
1	788	574	515	627	509	399	595
2	2050	1492	1338	1630	1322	1038	1548
3	2628	1823	1674	2094	1695	1331	1984
4	3942	2735	2511	3141	2543	1997	2976
5	5256	3646	3348	4188	3390	2662	3968

Proportions *hi*

groups	Denmark	Finland	France	Germany	Italy	Spain	UK
0	0.06	0.08	0.07	0.05	0.09	0.07	0.15
1	0.25	0.28	0.20	0.24	0.22	0.24	0.17
2	0.20	0.22	0.23	0.20	0.18	0.18	0.21
3	0.37	0.28	0.28	0.32	0.27	0.25	0.25
4	0.07	0.11	0.11	0.13	0.13	0.16	0.12
5	0.06	0.04	0.10	0.06	0.11	0.10	0.10

Gross and disposable monthly income in EUR

groups	Denmark		Finland		France		Germany		Italy		Spain		UK	
	Y_i	C_i	Y_i	C_i	Y_i	C_i	Y_i	C_i	Y_i	C_i	Y_i	C_i	Y_i	C_i
0	0	668	0	623	0	554	0	468	0	30	0	280	0	659
1	1432	1112	1109	969	907	918	1184	1019	686	569	749	917	1101	1149
2	2342	1523	1643	1242	1437	1186	1887	1306	1393	1068	1169	1326	1697	1491
3	3125	1858	2180	1537	2031	1584	2503	1620	1840	1359	1645	1629	2433	1903
4	4499	2424	3136	2027	2864	2216	3563	2229	2326	1650	2255	2054	3371	2575
5	6475	3650	4167	2670	4201	3084	5013	3180	3944	2697	3187	2711	4811	3595

Empirical evidence: Labor supply elasticity of Singles: a brief review

	Country	*Data*	*Selection*	*Extensive elasticity*	*Intensive elasticity*
Kleven and Kreiner (2006a, 2006b)	Denmark	ECHP 97-98	singles	0.45	0.2
Bargain and Orsini (2006)	Finland	IDS 97	single women	0.18 - 0.33	0.18 - 0.34
Bargain and Orsini (2006)	France	HBS 95	single women	0.04 - 0.07	0.08 - 0.14
Laroque and Salanie (2001)	France	Tax revenue 97	single women	0.36	
Bargain and Orsini (2006)	Germany	GSOEP 98	single women	0.08 - 0.15	0.09 - 0.18
Haan and Steiner (2005)	Germany	GSOEP 02	single women	0.01 - 0.09	0.02 - 0.24
			single men	0.06 - 0.19	0.09 - 0.28
Aaberge et al. (1998)	Italy	SHIW 1993	single women	0.06	0.10
			single men	0.08	0.11
Labeaga, Oliver and Spadaro (2007)	Spain	ECHP 95	singles	0.2	0.1
Blundell and MaCurdy (1999)	UK	FES 1980	singles	0.24	

Application: extensive elasticity

Participation elasticity in this model:

$$\chi_i = \frac{C_i - C_0}{h_i} \frac{\partial h_i}{\partial (C_i - C_0)}$$

- Classical participation elasticities from the literature correspond to 1% increase in Y_i rather than in $C_i - C_0 = Y_i - (T_i + C_0)$

- In most cases, $T_i + C_0 > 0$ so that $C_i - C_0$ increase by more than 1% and χ_i is overstated by usual estimates

- The inverse is true only when $T_i < -C_0$, i.e. when transfers to working poor are very large

Numerical application:

For low incomes [for group 0 to 2 (approx. 1st half)]: empirical estimated values in each countries. For high income = 0.

Application: intensive elasticity

Earnings (mobility) elasticity

$$\mu_i = \frac{C_i - C_{i-1}}{h_i} \frac{\partial h_i}{\partial (C_i - C_{i-1})}$$

Classical wage elasticity of labor supply

$$\varepsilon_i = \frac{1-\tau_i}{Y_i} \frac{\partial Y_i}{\partial (1-\tau_i)} \text{ with EMTR } \tau_i = \frac{T_i - T_{i-1}}{Y_i - Y_{i-1}}$$

Relating μ and ε:

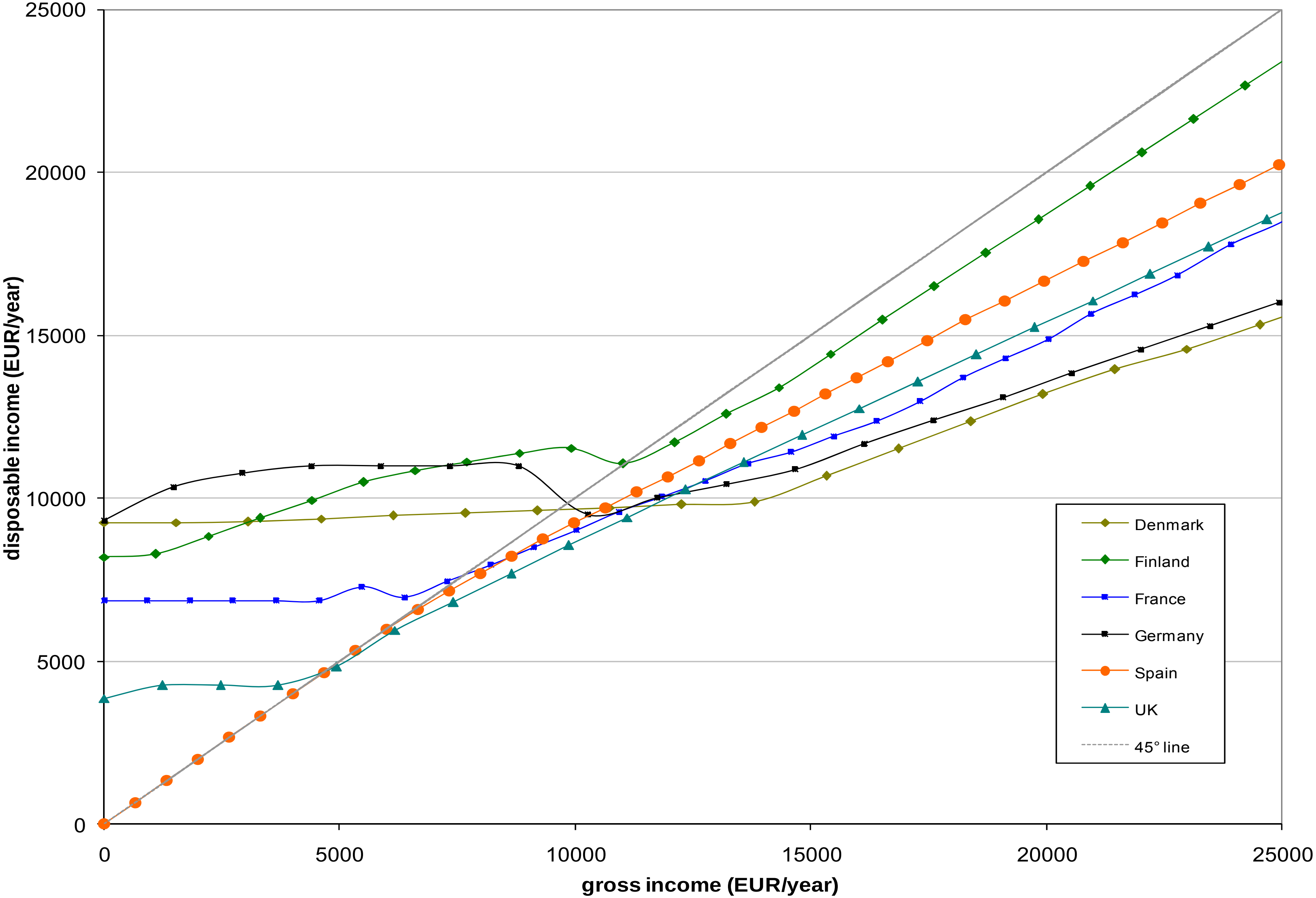
$$\mu_i (Y_i - Y_{i-1}) = \varepsilon_i Y_i$$

Numerical application (in terms of ε):

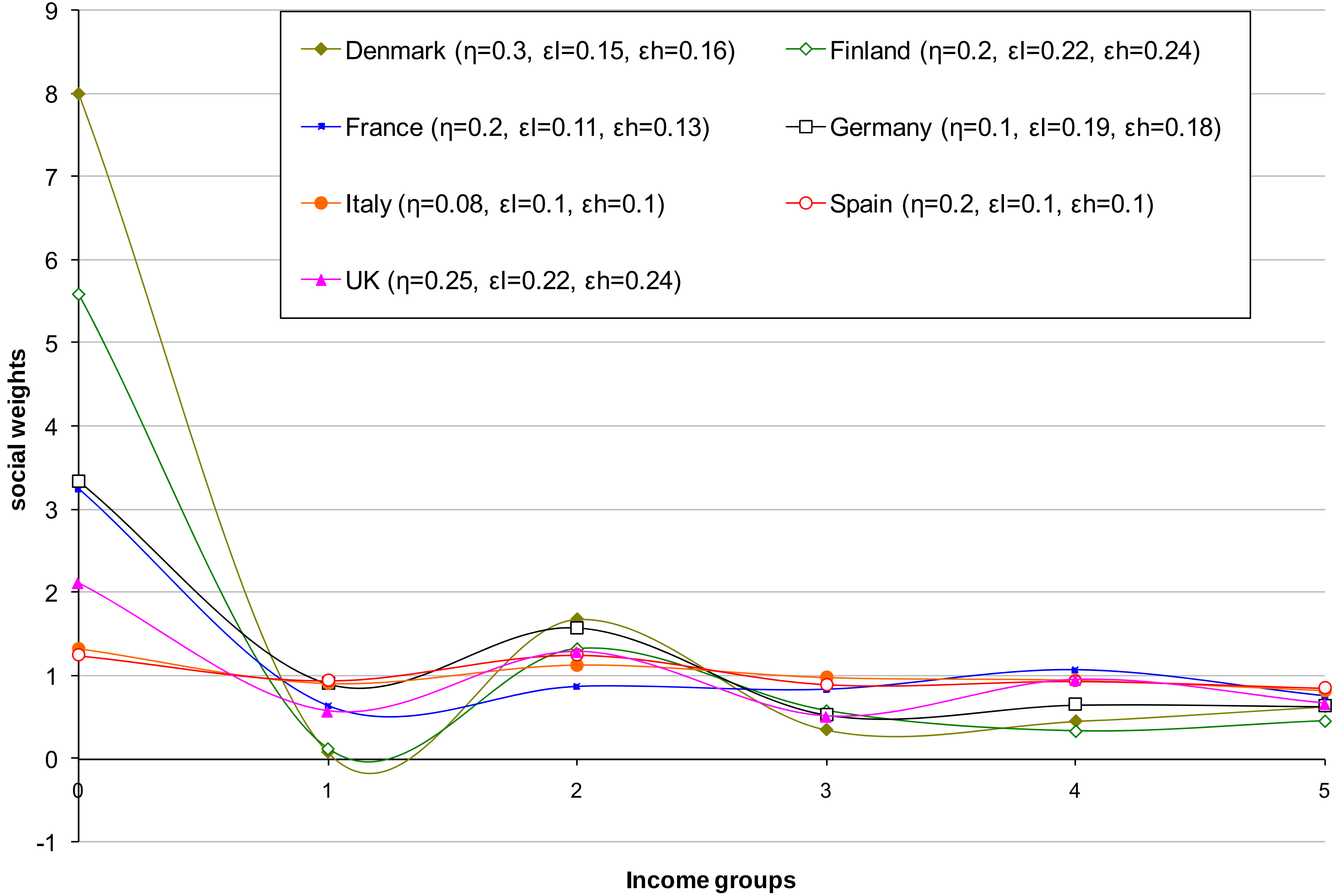
Empirical estimated values in each countries dividing in low [for group 0 to 2 (approx. 1st half)] and high income groups.

First intuition given by budget constraints for hypothetical households

Singles (low incomes)



Results: mixed model



Conclusions

Results in line with general intuition on welfare regimes

Equity concerns

- "flat" redistributive tastes in Southern Europe and to some extent in the UK
- generous SA translates into high weight on group 0 in Nordic countries, Germany and France; relatively flat for other groups = close to Rawlsian preferences

Efficiency concerns

- group 1: large distortion (high phase-out rate, esp. in Nordic countries) rationalised by lower social weights
- gap between weights on groups 0 and 1 even larger if high participation elasticity (=reason to accept distortions rationalised by social preferences)

More generally:

a) Result suggests that the redistribution systems in these countries are consistent with the hypothesis of an optimizing redistribution authority.

Limits

1. income taxes and benefits are only a very small part of the welfare state. In this sense, our contribution must be seen as a step toward the construction of formal theorizing allowing for better understanding the nature of welfare system and, eventually, to better define (if possible) ideal-typical models starting from the analysis of real welfare state.

2. it is natural to think that real world tax-benefit schedules result more from political economy forces than from the pursuit of some well defined social objective. Even though, deriving and comparing social welfare functions implicit in each national system provide a new way to compare countries' tastes for redistribution as embodied in tax-benefit systems.

3. The family dimension is completely missing in our analysis. This is an important shortcoming given that the role of the family, and in particular, the substitutability between state and families in providing protection against decommodification risks, is one of the pillars of the EA analysis.

Future work:

1. account for changes over time are desirable. In particular, recent trend toward EITC schemes in Europe may translate a change in social preferences, or the recognition of the disincentive effects.

2. more attention must be paid to the role of unemployment benefits and social contributions

3. treats social preferences as endogenously determined

4. dynamic dimension of the construction of the welfare state. Given the importance of the issue of intergenerational solidarity and the role of welfare state in his enhancement it would be interesting, for example, to try to fix a link between the ideal typology proposed in the "EA and others" literature and the theoretical literature on the optimal design of pensions system

Indirect Tax Reforms:
The Case of Spain

Alternative title of the presentation:

When the MATHS can say something about real world....

For example: **It is possible to implement Pareto improving indirect tax reforms in Spain?**

Related work:

- *India (Ahmad and Stern,1984)*
- *Norway (Christiansen and Jansen, 1978),*
- *Belgium (Decoster and Schokkaert, 1990),*
- *Canada (Cragg, 1991),*
- *Germany (Kaiser and Spahn, 1989),*
- *Italy (Brugiavini and Weber, 1988 and Liberati, 2001)*
- *Pakistan (Ahmad and Stern, 1991).*
- *Ireland (Madden, 1995)*
- *Greece (Kaplanoglou and Newbery, 2003)*

The theory (Diamond-Mirrlees):

- Production side:
  - Constant returns to scale.
  - Producer prices (*p*) are fixed.

- The government requires an amount T of resources collected via taxes (*t*) on goods.
  - Goods are indexed by *i*, *i=1…N.*

- Household factor incomes are fixed.
  - Consumer price: $q_i = p_i + t_i \longrightarrow dq = dt$.
  - Household are indexed by *h*, *h=1… H.*

The Problem:

The government solves the following maximization problem:

$$\max_{\{t_1, t_2, \ldots, t_n\}} \; W = W\left(V^1(q_1, \ldots q_n), V^2(q_1, \ldots, q_n) \ldots, V^H(q_1, \ldots, q_n)\right)$$

st. $T = \sum_{i=1}^{n} t_i X_i$ where $X_i = \sum_{h=1}^{H} x_i^h$

Solving with K.T.

$$L = W\left(V^1(q_1, \ldots q_n), V^2(q_1, \ldots, q_n) \ldots, V^H(q_1, \ldots, q_n)\right) + \lambda\left(\sum_{i=1}^{n} t_i X_i - \bar{T}\right)$$

Theoretical framework

Definition *Marginal Revenue Cost*: **cost at the margin in terms of revenue forgone when a tax is lowered so as to provide one extra unit in welfare**

$$\lambda_i = \frac{\partial R / \partial t_i}{\partial W / \partial t_i} \qquad \beta^h = \frac{\partial W}{\partial V^h}\frac{\partial V^h}{\partial m} \qquad \frac{\partial W}{\partial t_i} = -\sum_h \beta^h x_i^h$$

$$\frac{\partial R}{\partial t_i} = r_i = X_i + \sum_k t_k \frac{\partial X_k}{\partial t_i} = X_i\left(1+\sum_k \frac{q_k X_k}{q_i X_i}\cdot\frac{t_k}{q_k}\frac{q_i \partial X_k}{X_k \partial q_i}\right) = X_i\left(1+\sum_k \frac{\omega_k \tau_k \varepsilon_{ki}}{\omega_i}\right)$$

$$\lambda_i = \frac{\sum_h q_i x_i^h\left(1+\sum_h\sum_k \frac{\omega_k^h \tau_k \varepsilon_{ki}^h}{\omega_i^h}\right)}{\sum_h \beta^h q_i x_i^h}$$

τ is the tax on good *k* as a proportion of consumer price and ε is the uncompensated cross-price elasticity of good *k* with respect to good *i* f.o.c implies that MRC (λ) should be equal for all goods.

PARETO IMPROVING TAX REFORM PRINCIPLE:

if $MRC_i > MRC_s$ then higher t_i and lower t_s

Second order conditions are satisfied given the concavity of the Social Welfare Function.

Data Requirements

Four elements of data:

1. Household expenditure on goods (from a survey).
2. Demand derivatives (from a demand system estimation).
3. Effective taxes.
4. Welfare weights.

1. Spanish Household Budget Continuous Survey:

- Provided by the ‘Instituto Nacional de Estadística’.
- Available since 1984.
- It provides trimester and annual information about household resources and their expenditure on goods.
- The survey established the interview of households throughout 8 quarters.
- We used a longitudinal panel for year 1998. It has 9.891 observations and it represents 12.089.302 households and a population of 39.505.758.

2. Demand system estimation:

- Quadratic Almost Ideal Demand System (QUAIDS) for 16 commodities groups.
- The sample for the demand system estimation covers the period 1985-1997. (Change of methodology)
- Method of estimation: two stage least squares and non-linear instrumental variables.

3. Taxes:

- We use the effective taxes for each commodity group that was computed using a weighting sum of the different taxes for each good.

Some descriptive statistics and the effective taxes:

	Expenditure per equivalent adult				
Commodities	*mean*	*median*	*standard deviation*	*budget share*	*Effective tax (%)*
1.Food & non-alcoholic drinks	1821.72	1675.40	1028.28	0.1995	6.037
2.Alcoholic beverages	79.83	18.46	170.95	0.0077	16
3.Tobacco	179.20	86.43	248.13	0.0196	16
4.Clothing & footwear	729.01	554.75	689.14	0.0708	16
5.Housing expenditure	2242.79	1962.82	1410.55	0.2398	0
6.House keeping & services	918.65	668.06	924.31	0.0896	15.87
7.Fuel for housing	130.12	83.70	130.83	0.0139	16
8.Services	307.94	154.24	457.82	0.0290	2.46
9.Petrol	374.90	263.89	437.28	0.0362	16
10.Private transport services	290.24	156.15	392.70	0.0258	9.13
11.Public transport services	98.20	18.98	185.80	0.0094	7
12.Communications	202.65	166.41	166.95	0.0210	16
13.Leisure	1584.18	1080.10	1754.09	0.1362	6.98
14.Education	192.46	30.30	393.97	0.0163	12.64
15.Other non-durable goods	178.94	103.88	298.27	0.0183	11.56
16.Durable goods	1025.48	188.43	2606.16	0.0670	16

Data Requirements

4. Definition of welfare weights

Consider an additive iso-elastic social welfare function (Atkinson):

$$W = \sum_h U^h$$

where

$$\begin{cases} U^h(I^h) = \dfrac{k(I^h)^{1-e}}{1-e} & \text{if } e \geq 0, e \neq 1 \\ U^h(I^h) = k \log(I^h) & \text{if } e = 1 \end{cases}$$

$$U'(I^h) = \quad \beta^h = \left(\frac{I^1}{I^h} \right)^e$$

Where I^h is the equivalent income of household h

The Spanish Case: Some Results

Values of λ_i for different levels of inequality aversion.

The higher the rank the higher the taxes (ex. *Other non-durable goods*)

Commodities	Effective tax (%)	e=0	rank	e=1	rank	e=2	rank	e=5	rank
1.Food & non-alcoholic drinks	6.037	**0.9816**	12	**0.4025**	10	**0.2056**	6	**0.0722**	5
2.Alcoholic beverages	16	**1.1698**	5	**0.4435**	4	**0.2119**	5	**0.0668**	6
3.Tobacco	16	**1.3537**	2	**0.5499**	2	**0.2774**	2	**0.0953**	2
4.Clothing & footwear	16	**0.9604**	15	**0.3564**	15	**0.1661**	15	**0.0485**	13
5.Housing expenditure	0	**1.1183**	9	**0.4360**	6	**0.2170**	4	**0.0750**	4
6.House keeping & services	15.87	**1.0021**	11	**0.3701**	12	**0.1754**	11	**0.0551**	11
7.Fuel for housing	16	**1.1377**	7	**0.4417**	5	**0.2189**	3	**0.0761**	3
8.Services	2.46	**0.9811**	13	**0.3593**	14	**0.1669**	14	**0.0495**	12
9.Petrol	16	**1.1468**	6	**0.4265**	7	**0.1982**	8	**0.0578**	8
10.Private transport services	9.13	**1.2956**	3	**0.4573**	3	**0.2044**	7	**0.0563**	9
11.Public transport services	7	**0.9717**	14	**0.3601**	13	**0.1710**	12	**0.0553**	10
12.Communications	16	**1.1158**	10	**0.4168**	8	**0.1977**	9	**0.0613**	7
13.Leisure	6.98	**1.1213**	8	**0.3842**	11	**0.1676**	13	**0.0445**	15
14.Education	12.64	**0.8244**	16	**0.2700**	16	**0.1136**	16	**0.0282**	16
15.Other non-durable goods	11.56	**26.9443**	1	**10.3604**	1	**5.0223**	1	**1.6167**	1
16.Durable goods	16	**1.2179**	4	**0.4149**	9	**0.1785**	10	**0.0447**	14

Observation (1):

(a) The rank correlations suggest that the rankings, and thus the tax reform recommendations, show relatively strong sensitivity to the value of *e*. See for example the rank correlation among foods and leisure

- This result suggests that distributional considerations matter a lot in the ranking of goods.
- Indirect taxes seem to be a relatively efficient means of addressing distributional issues and reducing inequality in Spain.
- This is in contrast with previous results on Developed Countries.

Observation (2):

(b) Need to correct externalities. The consumption of goods such as alcohol, tobacco and petrol may give rise to social costs, which can be reduced by the imposition of corrective taxes.

Since we do not incorporate such effects in this model, it is possible that the observed rankings of these goods is explained by this factor.

Redistribution and Polarization Impact of the European Redistribution Architecture: an Analysis Using Microsimulation Techniques

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Introduction

Recent trends in economic and socio-demographic variables determined the rise of new demands of social protections that the actual Spanish model is unable to fully cover. For that reason, in the last years, the political and economic debate has been characterized by several proposals pushing for the reform of the Spanish welfare state.

- Spain belongs to what has been called “the Southern European (or Mediterranean)” welfare state regime (Esping Andersen 1990, 1999, Ferrera, 1996).

- Some reform proposals look toward a system more market oriented. Their reference model is the liberal type of welfare capitalism, which embodies individualism and the primacy of the market (for example, the UK system).

- There are also supporters of the Continental Europe Bismarkian social protection models. They push for the adoption of the so-called world of conservative corporatist welfare states, which is typified by a moderate level of decommodification (for example, the French system).

- Finally there are proposals of reforms in the spirit of the universalism observed in the Northern European countries: the so-called social-democratic world of welfare capitalism (for example, the Danish system).

	Social democracy	**Corporativist**	**Liberal**	**Southern-European**
Degree of decommodification	**Strong**	**Medium**	**Weak**	**Weak**
Ideological reference point	Universalism	Familiarism	Individual responsibility	Familiarism
Representative Countries	Denmark	Finland, Germany, France	UK	Spain, Italy

Whatever reform is implemented, it is important to have a clear picture of the impact it may cause on the economy.

In what follow we try to offer some elements of evidence of these effects. We will analyse the impact upon efficiency, income distribution and polarization of the replacement of the actual Spanish redistribution system with several European schemes (one for each “model”). In particular we simulate schemes similar to the ones enforced in France, UK and Denmark (corporatist, liberal and social-democratic respectively).

The efficiency, inequality and polarization analysis will be performed using behavioural microsimulation techniques.

The two main aims of the contribution are:

1) to offer some elements of clarification of the debate regarding the reforms of the welfare state in Spain by perform comparatives with other European welfare state regimes and

2) to show the potential of behavioural microsimulation models as powerful tools for the ex ante evaluation of public policies and their distributional and polarization impacts.

Definitions (Bourguignon and Spadaro, JoEI 2006):

•Microsimulation models allow simulating the effects of a policy on a sample of economic agents (individual, households, firms) at the individual level.

•Policy evaluation is based on representations of the economic environment of individual agents, their budget constraints and possibly their behavior.

•A policy simulation then consists of evaluating the consequences of a change in the economic environment induced by a policy reform on a vector of indicators of the activity or welfare for each individual agent in a sample of observations.

GladHispania is a microsimulation model of the Spanish Tax-Benefit system

- It is a:
  - Static
  - Partial equilibrium
  - With behavior
- It focuses on direct taxation (PIT and SS)
- It allows to simulate any change in those figures
- It uses the Spanish ECHP as a database

Simulated scenarios: The baseline is the 1999 Spanish tax-benefit system.

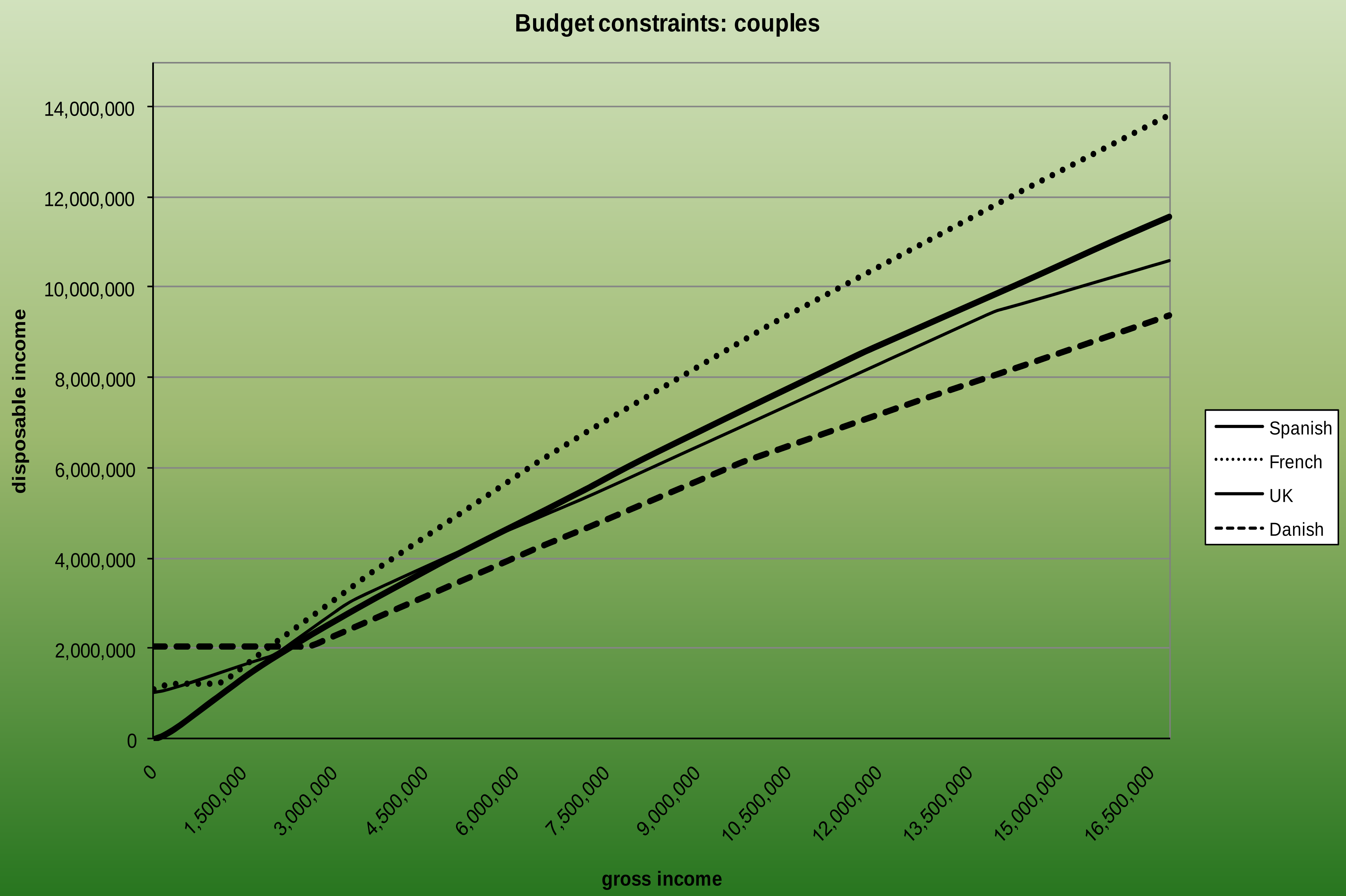
In order to simulate a system with the UK characteristics, we have simulated the following instruments: the income tax, the child benefit, the working families' tax credit and the income support.

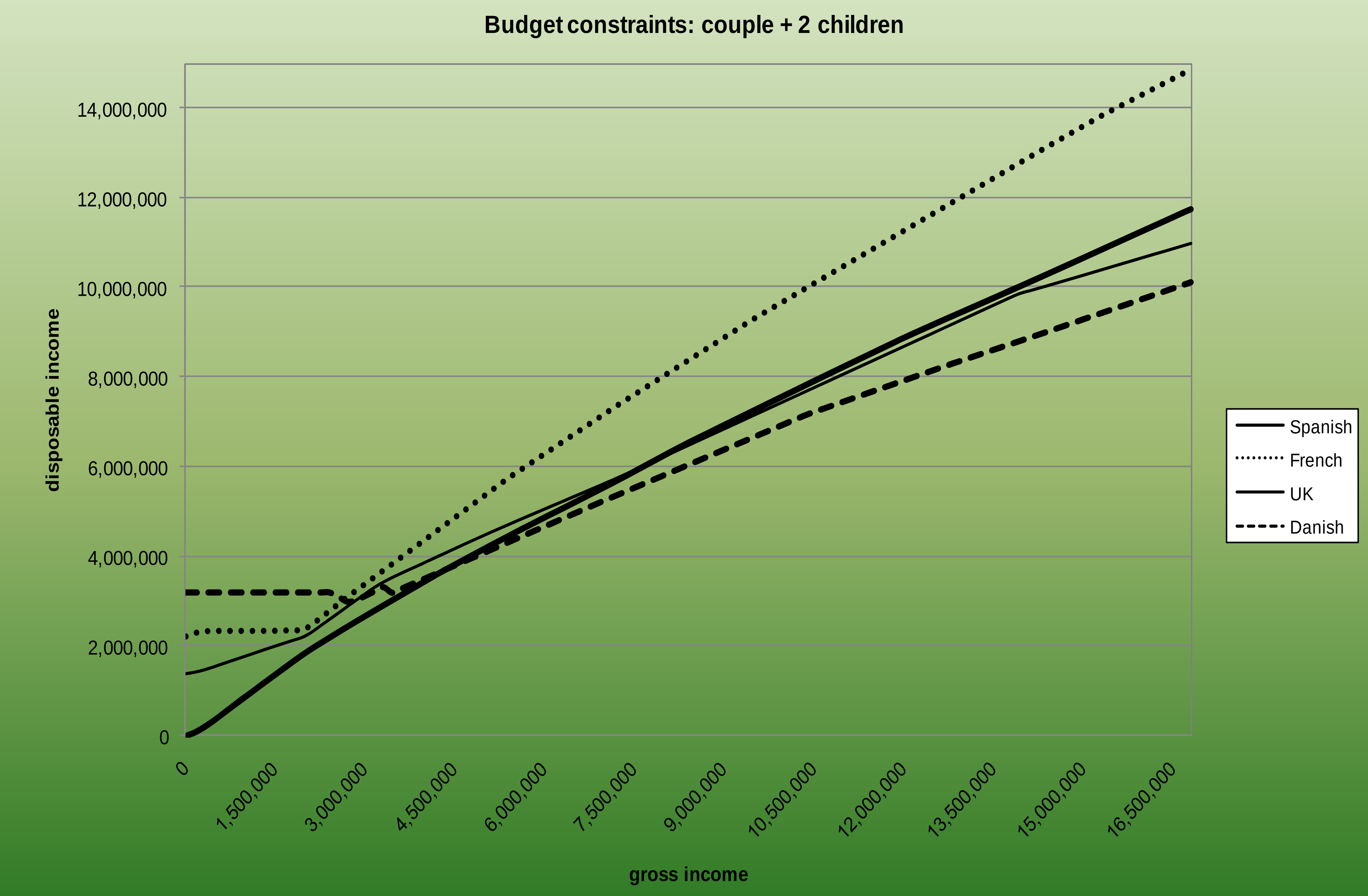
The French redistribution instruments that we model are: the "allocations familiales" , the "Revenue Minimum d'Insertion" , and the income tax.

The simulated social-democratic scenario is a simplification of the Danish one. In particular we model family allowances, social assistance and personal income taxation.

Spanish system[1]		**UK system**		**French system**[2]		**Danish system**	
up to	Tax rate	up to	Tax rate	up to	Tax rate	allowance	Tax rate
3,606	18.0%	2,956	10%	3,947	0.0%	4,481	6.25%
12,621	24.0%	48,284	22%	7,764	10.5%	23,867	6.00%
24,642	28.3%	over 48,284	40%	13,667	24.0%	37,148	15.00%
39,666	37.2%			22,129	33.0%		
66,111	45.0%			36,007	43.0%	4,481	31.75%[3]
over 66,111	48.0%			44,404	48.0%		
				over 44,404	54.0%		

Notes: (1) PIT tax rates schedules in 1999 are the same in 2001 (2) The tax schedule for France refers to the 1998 system. (3) In Denmark there is an important local tax that varies across regions. We have taken an average tax rate of 31.75%, which respect the total maximum marginal tax of 59%.


Budget constraints: couples
14,000,000
12,000,000
10,000,000
8,000,000
6,000,000
4,000,000
2,000,000
0
disposable income
0
1,500,000
3,000,000
4,500,000
6,000,000
7,500,000
9,000,000
10,500,000
12,000,000
13,500,000
15,000,000
16,500,000
gross income
Spanish
French
UK
Danish



Budget constraints: couple + 2 children
disposable income
14,000,000
12,000,000
10,000,000
8,000,000
6,000,000
4,000,000
2,000,000
0
0
1,500,000
3,000,000
4,500,000
6,000,000
7,500,000
9,000,000
10,500,000
12,000,000
13,500,000
15,000,000
16,500,000
gross income
Spanish
French
UK
Danish

Figure 2a: Singles

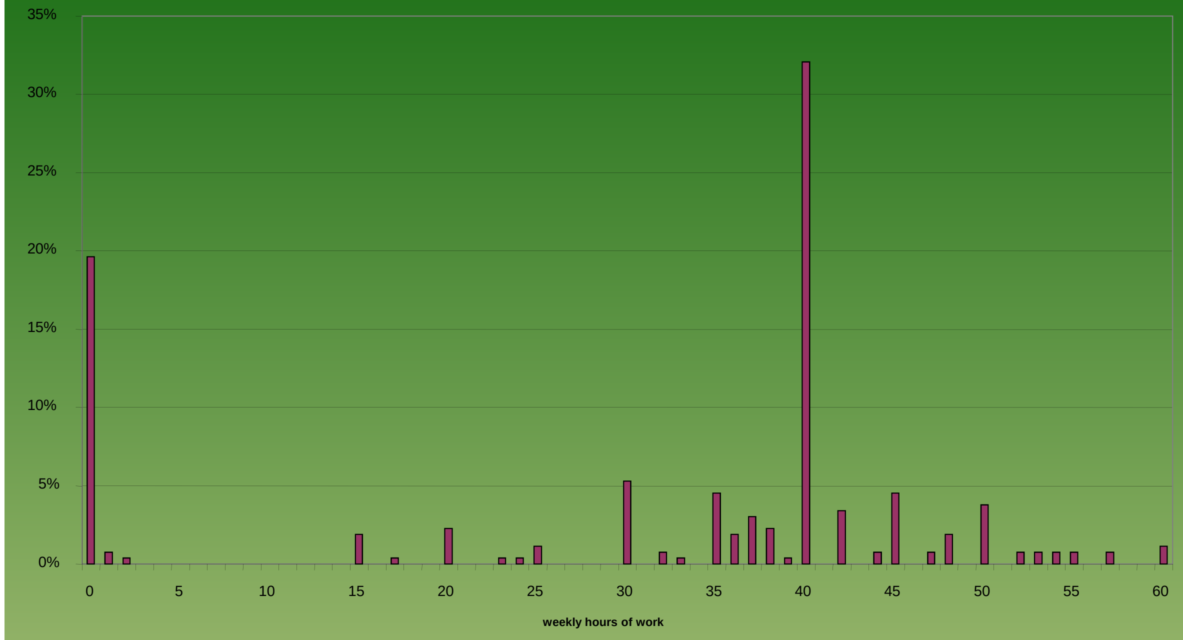


Figure 2b: Couples - Spouse

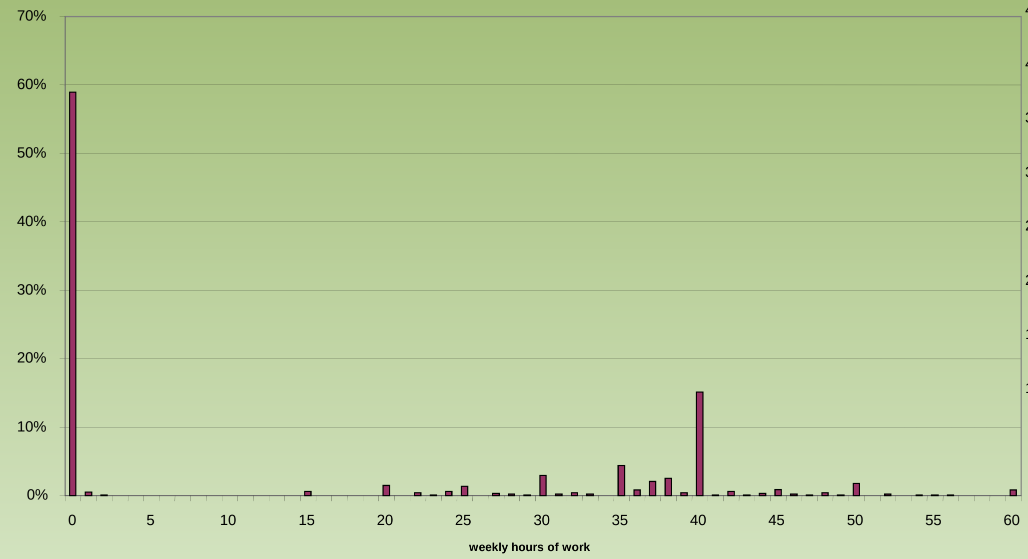
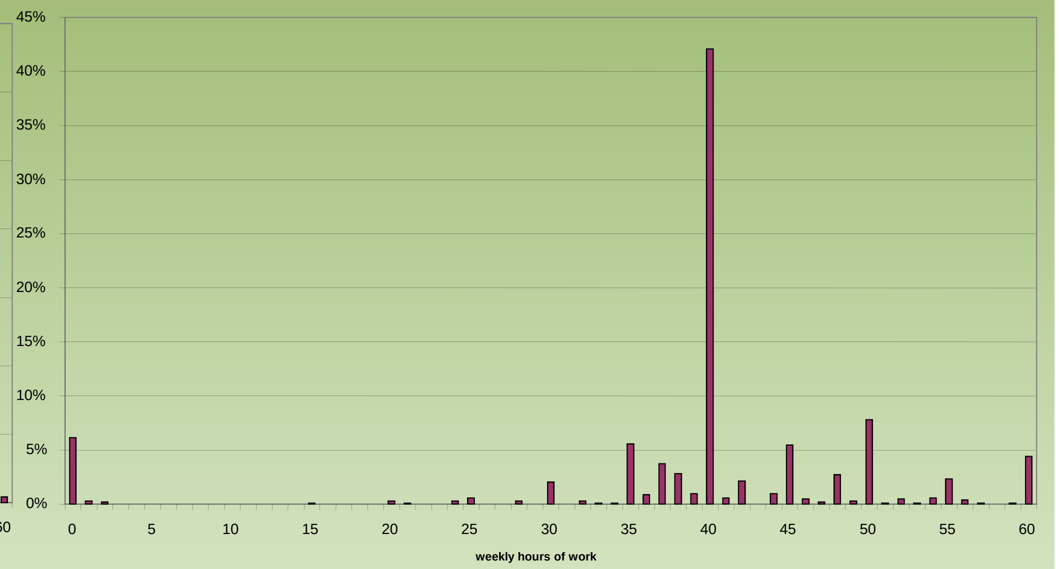


Figure 2b: Couples – Household head



Model specification and estimation: Aaberge et al. (1995) and van Soest (1995).

- Characteristics:
  - An utility function is estimated directly
  - There are a finite number of alternatives (K)

    $h_j = \{h_1, h_2, \ldots, h_K\}$
- Procedure:
  - There are *i* individuals and *j* alternatives
- We adopt the flexible quadratic utility function (as in Keane and Moffit, 1998, and Blundell *et al.*, 2000):

$$U^*(y, h, Z) = \alpha_{yy}\, y^2 + \alpha_{hh}\, h^2 + \alpha_{yh}\, yh + \beta_y(Z)\, y + \beta_h\,(Z)\, h + \varepsilon_{hi}$$

for the singles subsample, and

$$U^*(y, h_h, h_c, Z_h, Z_c, Z) = \alpha_{yy} y^2 + \alpha_{h_h h_h} h_h^2 + \alpha_{h_c h_c} h_c^2 + \alpha_{y h_h} y h_h + \alpha_{y h_c} y h_c + \alpha_{h_h h_c} h_h h_c + \beta_y y + \\ + \beta_{h_h} h_h + \beta_{h_c} h_c + \varepsilon_{h_h h_c}$$

for couples.

  - *y = disposable income – fixed costs*
  - It is assumed that individuals choose the alternative that maximizes his utility

Model specification and estimation: Log-likelihood

- We assume that ε follows a Weibull distribution

$$P_{ik} = \Pr(V_{ik} \geq V_{ij}, \forall j = 1,\ldots, J) = \frac{\exp\left[U(y_{ik,}L_k, X_i; v_k)\right]}{\sum_{i=1}^{J} \exp\left[U(y_{ij,}L_j, X_i; v_j)\right]}$$

- The log-likelihood function:

$$\ln L = \sum_{i=1}^{N} \sum_{j=1}^{J} d_j \ln(P_{ij})$$

- This is the McFadden or conditional logit model

Singles estimation

Variable	Coefficient	Standard error
$Income^2$	-0.41	0.50
Hours of $leisure^2$	-236.95	32.44
Income x Hours of leisure	29.06	5.81
Income	-25.54	6.77
x Age	0.50	0.25
x Education	0.04	0.84
x Children	0.19	0.16
Hours of leisure	458.94	65.24
x Age	-0.49	1.53
x Educ1	-4.19	3.93
x Educ2	0.39	2.89
Fixed costs	2.40	0.50
Number of observations	259	
Log likelihood	-273.84	

Couples estimation

Variable	Coefficient	Standard Error
$Income^2$	-0.71	0.16
Hours of leisure of the household's $head^2$	-83.69	6.30
Hours of leisure of the $spouse^2$	91.98	8.01
Income x Hours of leisure of the household's head	-2.74	1.51
Income x Hours of leisure of the spouse	-1.69	1.01
Hours of leisure of the household's head x Hours of leisure of the spouse	-44.8	7.98
Income	8.20	2.37
x Age of the household's head	-0.60	0.48
x Age of the spouse	1.54	0.55
x Age of the spouse ^{2}	-0.63	0.19
Hours of leisure of the household's head	197.53	17.25
x Education of the household's head	-5.68	1.81
x Age of the household's head	2.19	0.67
Hours of leisure of the spouse	-117.38	17.65
x Education of the spouse	-11.1	1.20
x Age of the spouse	2.02	0.61
x 1(one dependent child)	2.82	0.95
x 1(two or more dependent children)	5.05	0.90
Fixed costs	-0.35	0.26
Number of observations	1024	
Log likelihood	-1553.81	

Results: Efficiency

		Spanish system									
Combination of working hours (household head_spouse)		0_0	0_25	0_40	40_0	40_25	40_40	50_0	50_25	50_40	total
Danish system	0_0	**0.62**	0.00	0.00	0.10	0.00	**0.10**	0.31	0.00	0.00	1.14
	0_25	0.00	**0.10**	0.00	**0.21**	0.00	0.00	0.00	0.00	0.10	0.41
	0_40	0.10	0.00	**3.52**	0.31	**0.31**	**0.41**	0.10	0.10	0.00	4.86
	40_0	0.00	0.00	0.00	**36.71**	0.00	0.10	0.21	0.10	0.10	37.23
	40_25	0.00	0.00	0.00	0.00	**6.72**	0.00	0.00	0.00	0.10	6.83
	40_40	0.00	0.00	0.10	0.00	0.00	**17.37**	0.10	0.00	0.00	17.58
	50_0	0.00	0.00	0.00	0.00	0.00	0.00	**22.23**	0.00	0.00	22.23
	50_25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**2.28**	0.00	2.28
	50_40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**7.45**	7.45
	total	0.72	0.10	3.62	37.33	7.03	17.99	22.96	2.48	7.76	**100.00**

		Spanish system									
Combination of working hours (household head_spouse)		0_0	0_25	0_40	40_0	40_25	40_40	50_0	50_25	50_40	total
French system	0_0	**0.72**	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.83
	0_25	0.00	**0.10**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
	0_40	0.00	0.00	**3.62**	0.10	0.00	0.00	0.00	0.00	0.00	3.72
	40_0	0.00	0.00	0.00	**36.40**	0.00	0.00	0.00	0.10	0.00	36.50
	40_25	0.00	0.00	0.00	0.00	**6.83**	0.00	0.10	0.00	0.00	6.93
	40_40	0.00	0.00	0.00	0.00	0.00	**17.79**	0.10	0.00	0.00	17.89
	50_0	0.00	0.00	0.00	0.83	0.00	0.10	**22.75**	0.00	0.00	23.68
	50_25	0.00	0.00	0.00	0.00	0.10	0.00	0.00	**2.38**	0.00	2.48
	50_40	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	**7.76**	7.86
	total	0.72	0.10	3.62	37.33	7.03	17.99	22.96	2.48	7.76	**100.00**

		Spanish system									
Combination of working hours (household head_spouse)		0_0	0_25	0_40	40_0	40_25	40_40	50_0	50_25	50_40	total
UK system	0_0	**0.72**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.72
	0_25	0.00	**0.10**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
	0_40	0.00	0.00	**3.62**	0.21	0.00	0.10	0.00	0.00	0.00	3.93
	40_0	0.00	0.00	0.00	**37.13**	0.00	0.00	0.31	0.10	0.10	37.64
	40_25	0.00	0.00	0.00	0.00	**7.03**	0.00	0.00	0.00	0.10	7.14
	40_40	0.00	0.00	0.00	0.00	0.00	**17.89**	0.00	0.00	0.00	17.89
	50_0	0.00	0.00	0.00	0.00	0.00	0.00	**22.65**	0.00	0.00	22.65
	50_25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**2.38**	0.00	2.38
	50_40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**7.55**	7.55
	total	0.72	0.10	3.62	37.33	7.03	17.99	22.96	2.48	7.76	**100.00**

Results: Efficiency

With such evidence, two points should be stressed:

1. the majority of households are on the diagonal, which implies that they do not alter their labour supply;
2. the higher the marginal tax rate, the greater are the labour supply effects.

It is also interesting to look at changes in labour supply behaviour of spouses. It must be noted that, in around 95% of the sample, they are women. It is clear that female labour supply and participation is stimulated under the Danish system. 0.53% of women increase their labour supply after the reform (Danish system) against 0.1% under the French system and -0.11% under the UK system.

The measure of polarization

According the axiomatic discussion in Duclos, Esteban and Ray (2004) the functional form of $T(i, a)$ is chosen such that

$$P_\alpha(f) \equiv \iint f(x)^{1+\alpha} f(y)|x-y|dydx,$$

where α is arbitrary chosen such that $\alpha \in [.25,1]$.[6]

Finally, considering any distribution function F with associated density f and mean μ, the polarization index can be written as

$$P_\alpha(F) = \int_y f(y)^\alpha a(y)dF(y),$$

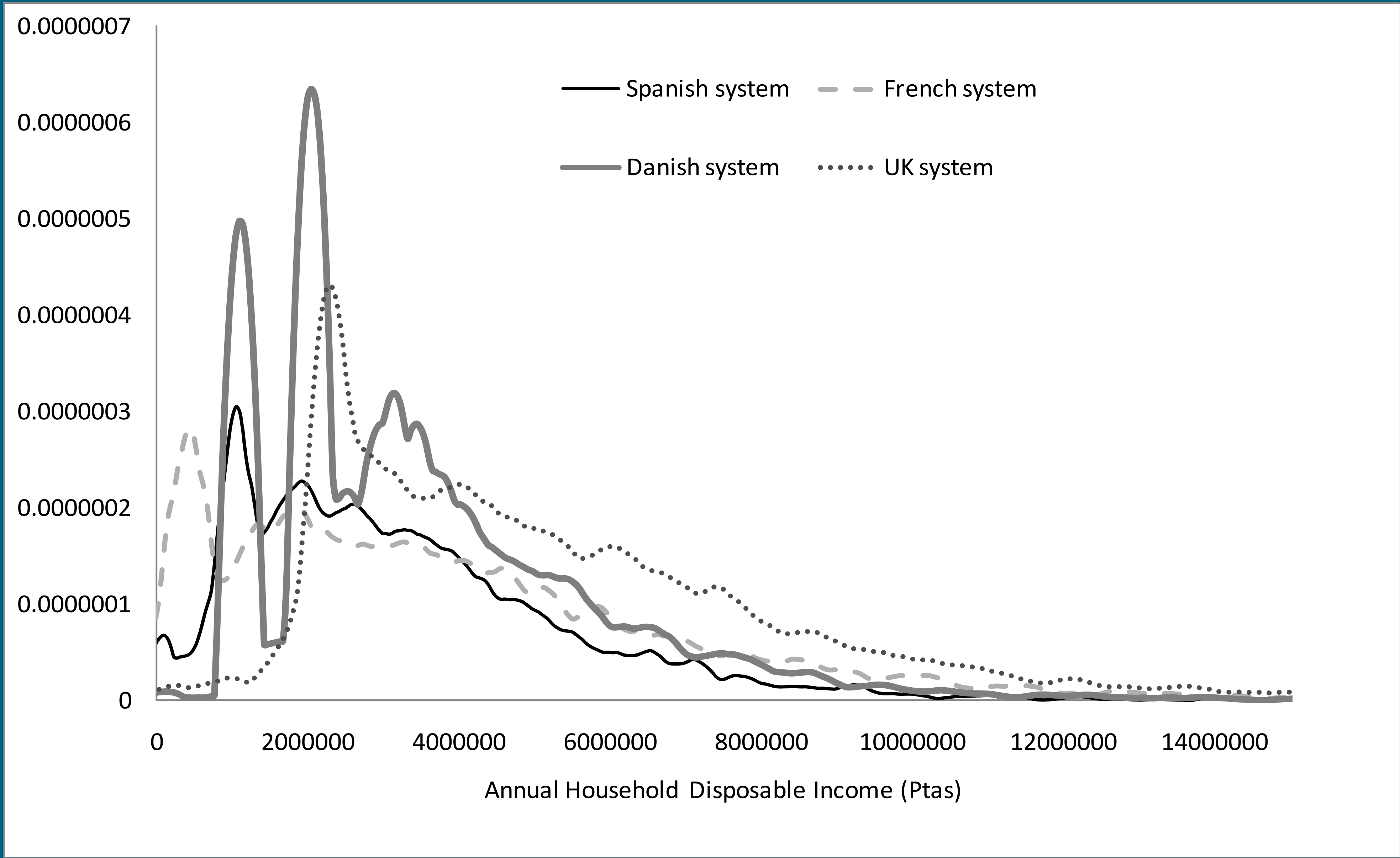
with $a(y) = \mu + y(2F(y) - 1) - 2\int_{-\infty}^{y} xdF(x)$.

An individual located at x in the distribution of the characteristic feels alienation with respect to another individual located at y according to their distance $|x-y|$ and identifies with the group depending on the density at $x, f(x)$.

A hybrid measure of polarization in which both identification and alienation may depend on income and other characteristics is

$$P^*(\boldsymbol{F}) = \sum_{j=1}^{M} \sum_{k \neq j} \iint_{x\,y} f_j(x)^\alpha |x-y| dF_j(x) dF_j(y).$$

Results: Inequality and Polarization



Results: Inequality and Polarization

Table 8. Inequality and Polarization indexes

	Gini	alpha = 0.25	alpha = 0.5	alpha = 0.75	alpha = 1
Spanish system	0.3604 (0.0053)	0.2735 (0.0031)	0.2206 (0.0022)	0.1845 (0.0018)	0.1577 (0.0018)
UK system	0.3084 (0.0037)	0.2463 (0.0024)	0.2086 (0.0018)	0.1831 (0.0016)	0.1644 (0.0017)
French system	0.3373 (0.0044)	0.2631 (0.0027)	0.2172 (0.0020)	0.1854 (0.0017)	0.1616 (0.0016)
Danish system	0.2230 (0.0040)	0.1982 (0.0027)	0.1901 (0.0024)	0.1909 (0.0027)	0.1975 (0.0034)

Table 11. Polarization by age class					
		Spanish system	Danish system	French system	UK system
Gini	Less than 35	0.3291 (0.0132)	0.1811 (0.0094)	0.2731 (0.0073)	0.2643 (0.0087)
	Between 35 and 60	0.3467 (0.0073)	0.2193 (0.0057)	0.3120 (0.0060)	0.2975 (0.0050)
	More than 60	0.3680 (0.0081)	0.2272 (0.0057)	0.3733 (0.0071)	0.3236 (0.0070)
alpha=.5	Less than 35	0.2125 (0.0064)	0.1615 (0.0064)	0.1983 (0.0042)	0.1881 (0.0043)
	Between 35 and 60	0.2143 (0.0031)	0.1792 (0.0032)	0.2066 (0.0028)	0.2003 (0.0023)
	More than 60	0.2372 (0.0042)	0.2447 (0.0052)	0.2478 (0.0041)	0.2422 (0.0046)
alpha=1	Less than 35	0.1533 (0.0046)	0.1681 (0.0066)	0.1619 (0.0045)	0.1514 (0.0034)
	Between 35 and 60	0.1541 (0.0024)	0.1764 (0.0035)	0.1599 (0.0023)	0.1559 (0.0019)
	More than 60	0.1866 (0.0045)	0.3643 (0.0129)	0.1968 (0.0047)	0.2303 (0.0069)

Table 12. Polarization by gender for singles (no children)					
		Spanish system	Danish system	French system	UK system
Gini	Couples	0.3478 (0.0056)	0.2141 (0.0043)	0.3228 (0.0047)	0.2981 (0.0040)
	Males	0.4021 (0.0161)	0.2373 (0.0134)	0.3801 (0.0154)	0.3427 (0.0135)
	Females	0.4275 (0.0245)	0.1620 (0.0228)	0.4237 (0.0255)	0.3088 (0.0274)
alpha=.5	Couples	0.2157 (0.0023)	0.1868 (0.0026)	0.2123 (0.0021)	0.2034 (0.0019)
	Males	0.2467 (0.0093)	0.2364 (0.0127)	0.2481 (0.0102)	0.2328 (0.0088)
	Females	0.2982 (0.0216)	0.2724 (0.0394)	0.3336 (0.0252)	0.2811 (0.0283)
alpha=1	Couples	0.1566 (0.0019)	0.2027 (0.0041)	0.1617 (0.0018)	0.1615 (0.0017)
	Males	0.1750 (0.0076)	0.2974 (0.0206)	0.1860 (0.0083)	0.1888 (0.0098)
	Females	0.3084 (0.0297)	0.7559 (0.1099)	0.3927 (0.0380)	0.4082 (0.0471)

Table 13. Polarization by education

		Spanish system	Danish system	French system	UK system
Gini	Graduate	0.3139 (0.0131)	0.2550 (0.0106)	0.3025 (0.0104)	0.2750 (0.0087)
	Secondary	0.2988 (0.0116)	0.2029 (0.0080)	0.2792 (0.0092)	0.2631 (0.0088)
	Primary	0.3304 (0.0052)	0.1913 (0.0036)	0.3049 (0.0040)	0.2814 (0.0040)
alpha=.5	Graduate	0.2061 (0.0066)	0.1897 (0.0060)	0.2041 (0.0054)	0.1912 (0.0043)
	Secondary	0.2010 (0.0056)	0.1804 (0.0060)	0.1981 (0.0050)	0.1903 (0.0043)
	Primary	0.2108 (0.0021)	0.1846 (0.0026)	0.2071 (0.0019)	0.2004 (0.0019)
alpha=1	Graduate	0.1557 (0.0048)	0.1609 (0.0047)	0.1546 (0.0036)	0.1487 (0.0029)
	Secondary	0.1482 (0.0032)	0.1799 (0.0076)	0.1527 (0.0035)	0.1515 (0.0030)
	Primary	0.1529 (0.0017)	0.2165 (0.0047)	0.1568 (0.0015)	0.1647 (0.0020)

Table 14. Polarization by working position

		Spanish system	Danish system	French system	UK system
Gini	Other positions	0.3696 (0.0064)	0.2087 (0.0033)	0.3444 (0.0056)	0.3057 (0.0045)
	Employee	0.2851 (0.0082)	0.2134 (0.0051)	0.2788 (0.0069)	0.2489 (0.0044)
	Self employed	0.3755 (0.0183)	0.1918 (0.0101)	0.2779 (0.0132)	0.2927 (0.0095)
alpha=.5	Other positions	0.2286 (0.0029)	0.2059 (0.0028)	0.2280 (0.0029)	0.2163 (0.0025)
	Employee	0.1950 (0.0040)	0.1737 (0.0028)	0.1940 (0.0034)	0.1805 (0.0020)
	Self employed	0.2324 (0.0097)	0.1739 (0.0070)	0.1981 (0.0077)	0.1992 (0.0046)
alpha=1	Other positions	0.1681 (0.0028)	0.2565 (0.0062)	0.1763 (0.0028)	0.1866 (0.0032)
	Employee	0.1585 (0.0034)	0.1694 (0.0031)	0.1574 (0.0027)	0.1510 (0.0015)
	Self employed	0.1670 (0.0073)	0.1939 (0.0085)	0.1669 (0.0066)	0.1629 (0.0043)

The results show that the scenarios simulated have little impact on the efficiency of the economy (as measured by labour supply effects).

Concerning inequality the Danish system is the best one. To a lower degree, a result in this same direction can be achieved also adopting the French and UK systems.

However, when we take into consideration income polarization the situation is much less clear:

The results of our analysis in term of polarization show how important it is to consider not only redistribution effects. The decision of which reform should be implemented appears not so easy as if we were considering only income inequality.

Question: how much a policy maker should weight this additional polarization information?