Government Beliefs about Individual Preferences: an Application of the Optimal Tax Inverse Problem[1]

Amedeo Spadaro
Paris School of Economics
and
Universitat de les Illes Balears, Palma de Mallorca

Abstract:

In this paper, using the inversion of the optimal income tax problem approach, we recover the government beliefs about individual preferences starting from the observed distribution of gross and disposable income within a population and from the observed marginal tax rates as computed in standard microsimulation models. A detailed application is given in the case of France and Spain. The results show that, in the two countries analyzed, the government beliefs about the value of elasticity all along the income range are not uniform. Poor and rich people react less than the middle class. This result highlights the limit of using estimations of average labor supply elasticity in the analysis of fiscal reforms. We show also that the heterogeneity of the treatment of household size, implicit in the tax systems analyzed, influences dramatically the elasticity calculated. This highlights the relevance of defining a theoretical framework allowing for the explicit treatment of multidimensional optimal redistribution. The approach implemented in this paper provides an alternative way of reading marginal tax rates calculations routinely provided by tax-benefit models. In that framework, the issue of the optimality of an existing tax-benefit system may be analyzed by considering whether the individual utility function associated with that system satisfies elementary properties.

Keywords: Optimal Income Tax, Microsimulation, Labor Supply

JEL Classifications: H21, C63
January 2008
Preliminary and incomplete draft. Do not quote.

[1] I acknowledge Salvador Balle, Jim Mirrlees, the participants to the Conferences of Marseille 2004, Beijing 2004, Paris 2008 and seminar participants at University Pompeu Fabra, PSE and FEDEA for their helpful comments. All errors remain our own. I acknowledge financial support of Spanish Government (SEJ2005-08783-C04-03/ECON) and of French Government (ANR BLAN06-2_139446).
Correspondence: Amedeo Spadaro, Paris School of Economics, 48 bd Jourdan 75014 Paris, France, email: spadaro@pse.ens.fr

Introduction

Several attempts were recently made at analyzing existing redistribution systems in various countries within the framework of optimal income taxation theory. The question asked in that literature is whether it is possible to justify the most salient features of existing systems by some optimal tax argument. For instance, under what condition would it be optimal for the marginal tax rate curve to be U-shaped? (see Diamond (1998) and Saez (2001) for the US and Salanié (1998) for France). Or could it be optimal to have close to 100 per cent effective marginal tax rates at the bottom of the distribution as in some minimum income programs ? [Piketty (1997), D'Autume (2001), Bourguignon and Spadaro (2000a), Choné and Laroque (2001)]. Such questions were already addressed in the early optimal taxation literature and in particular in Mirrlees (1971, 1986) but the exercise may now seem more relevant because of the possibility of relying on large and well documented micro data sets and simulation models rather than on hypothetical statistical distributions.

The optimal income tax model[2] defines the tax schedule as a function of three key ingredients: the distribution of productivity or wage rate in the population, individual preferences for consumption and leisure, and the shape of the social welfare function. The optimal tax schedule then maximizes this social welfare functions under some government budget constraint and the constraints born from the labor supply behavior of agents. Practically, applied optimal taxation models describe the optimal tax schedule through marginal tax rates expressed as a function of the distribution of observed hourly wages, the income and wage elasticity of labor supply estimated on observed hours of work and some arbitrary specification of the social welfare function. Comparing the results of this computation with the shape of the actual effective marginal tax then suggests ways to improve redistribution systems for given assumptions on the social welfare function.

A somewhat different approach based on optimal inverse methods was explored in Bourguignon and Spadaro (2000b, 2002). Instead of solving the optimal income taxation model with respect to marginal tax rates and comparing them with actual ones, the idea was to identify the social welfare function that makes actual marginal tax rates optimal. In other words, the objective was to identify the social preferences *revealed* by actual marginal tax rates given a distribution of individual productivities and labor/consumption preferences. Practically this was done into two steps. First, implicit household productivities were estimated on the basis of observed gross labor incomes, the budget constraint implied by the existing tax-benefit system and arbitrary labor/consumption preferences. This procedure was justified by the belief that households, rather than active individuals should be taken as the welfare units. This made necessary to consider the joint labor supply and participation of couples as the utility maximizing decision of a single rational agent, hence the need to use some kind of implicit wage rate, or 'productivity', for the couple. Second, an 'optimal inverse' approach (see Kurz, 1968) was used to recover the social welfare function compatible with the assumption that the actual tax-benefit system was indeed optimal, given the preceding distribution of productivities and the arbitrary labor/consumption preferences.

This approach may be seen as the dual of the usual one. Wondering about the optimality of an actual redistribution system generally consists of comparing an optimal effective marginal tax rate schedule derived from some 'reasonable' arbitrary social welfare function with the actual one. In the preceding case, it consists of checking whether the social welfare function consistent with the actual redistribution schedule and arbitrary individual labor/consumption preferences is 'well behaved', that is whether the marginal social welfare is everywhere

[2] See for instance Atkinson and Stiglitz (1980) or Tuomala (1990).

positive and decreasing. This may also be seen as an original way of 'reading' the average and marginal net tax curves that are commonly used to describe a redistribution system. In effect, observed curves are translated into social welfare functions. Comparing two redistribution systems or analyzing the reform of an existing system may then be made directly in terms of social welfare.

This methodology was applied to redistribution schemes in operation in France, Spain and the UK in 1995 (Bourguignon and Spadaro 2000b), under the assumption of quasi-linear consumption/labor preferences. Revealed marginal social welfare curves were found to be well-behaved when the wage elasticity of labor supply was assumed to be low (0.1). However, marginal social welfare turned out to be negative at the very top of the distribution when the labor supply elasticity was assumed to be around the average of econometric estimates available in the literature (0.5). This feature was present in the three countries, although more pronounced in the case of France. It suggested that redistribution authorities could be non-Paretian (Bourguignon and Spadaro 2002) or inefficient because not maximizing social welfare, in both case presumably as the result of political economy phenomena.

Assumptions on labor-supply elasticity clearly play a key role in the preceding conclusion. Governments are revealed to have well-behaved social preferences when they believe the labor supply response to changes in net wages is low and are non-Paretian in the opposite case. This suggests another way of defining the dual of the standard optimal income tax model. Instead of deriving the social welfare function consistent with the assumption that the actual redistribution system is optimal and with arbitrary labor supply elasticity, it is possible to derive the labor supply elasticity that is consistent with the same optimality assumption but with an arbitrary social welfare function. Of course, this requires assuming that individual consumption/labor preferences may vary with household productivity. The problem is then to know whether this variation seems reasonable or not.

This is the approach taken in this paper. The 'optimal inverse' problem solved here offers still another way of reading marginal tax rate curves routinely provided by tax-benefit models. Instead of reinterpreting these curves in terms of what they imply for marginal social welfare, as before, the idea is now to show the labor-supply elasticity that their presumed optimality imply at each level of household productivity. The question of the optimality of the existing tax-benefit system thus appears under another form, namely whether the implied values of the labor supply elasticity at various levels of productivity make sense or not.

The structure of the paper is as follows. Section 1 recalls the optimal taxation model and derives the duality relationship between the effective marginal tax rate schedule and the individual utility function. This is done in the case where social preferences are assumed to be of the *Hyperbolic Absolute Risk Aversion (HARA)* type and individual preferences are restricted to be quasi-linear with respect to consumption. The second section presents the way this relationship was implemented empirically. Section 3 discusses the labor supply elasticity estimations in the econometric literature. The fourth section is devoted to the data and the microsimulation model used in the empirical part of the paper. Section 5 analyses the results obtained in the case of France and Spain under alternative assumptions about the degree of social aversion to inequality. Section 6 concludes.

1. The Theoretical Framework

Mirrlees optimal income tax (or redistribution) model, in its canonical form, can be stated as follows.

$$\text{Max}_{T()} \int_{w_0}^{A} H[V[w,T()]].f(w).dw \qquad (1.1)$$

$$\text{under contraints:} \quad (C^*,L^*) = \text{Argmax}[U(C,L);\ C = wL - T(wL),\ L \geq 0] \qquad (1.2)$$

$$V[w,T()] = U(C^*,L^*) \qquad (1.3)$$

$$\int_{w_0}^{A} T(wL^*).f(w).dw \geq B \qquad (1.4)$$

In this optimization program, the function *U(C,L)*, supposed increasing in consumption, decreasing in labor supply and quasi-concave in both arguments, represents the preferences of an agent between all the combinations of the real expenses of consumption *(C)* and work *(L).* The combination *(C*, L*)* is the preferred combination, under the budget constraint he/she confronts; *w* is the work unit income, that is to say the wage rate, if we suppose that *L* measures only the work duration or the " productivity " of an agent in a more general case. *T(wL)* is the tax paid. It is supposed to be only a function of the observed total income. *V( )* is the utility level obtained effectively by the agent. Therefore it depends on his productivity and on the redistribution system *T(wL).* The distribution of productivities *f(w)* in the population is defined within the interval (w_0, A). Finally, *B* is the budget that the government has to finance. From this point of view, the government is supposed to maximize the total social value of the individual utilities respect to the redistribution function *T(wL).* The relation between the private value and the social value of the individual utility is represented by the function *H(V)*, supposed to be concave.

The concavity of *H(V)* means that the government would like to redistribute part of the income of those who have a higher productivity and income to the people with low productivities. A way of obtaining this result is by increasing the tax *T(wL)* according to income. But if it increases too quickly, the labor supply L* can decrease and the total amount to be redistributed can then being insufficient after considering the government budgetary constraint. The trade off between efficiency – in other words a high level of labor supply and monetary income – and equity, or redistribution, constitutes then the heart of the model. Under this general form, we can see that the optimal redistribution, represented by *T(wL)* is a function of the individual labor supply behavior (as it proceeds from the preferences *U(C,L)*), of the distribution of the productivities *f(w)* and, finally, of the social welfare function *H(V).*

The general solution of this problem is complex[3]. It is therefore rarely implemented without restrictions on individual preferences. A particular case, which has recently received a lot of attention, is the one where utility is separable with respect to consumption and work. A frequently used class of separable functions is the quasi linear in consumption:

$$U(C,L) = C - B(L) \qquad (2)$$

where *B(L)* is a function decreasing and quasi-concave. It is easy to see that labor supply income elasticity is 0.

[3] See Atkinson and Stiglitz (1980).

With this particular specification of the preferences, we can easily show that the optimal marginal tax rate *t(w)* of an agent whose productivity is *w*, is given by :

$$\frac{t(w)}{1-t(w)}=\left[1+\frac{B''(L)L}{B'(L)}\right]\left[\frac{1-F(w)}{w.f(w)}\right].\{1-S(w)\} \qquad (3)$$ [5]

with $S(w)=\int_{w}^{A} H'(V)f(x)dx/[(1-F(w)]$ (4)

where *F( )* is the cumulative associated with *f( )* and *S(w)* is the average marginal social value of the income of all the agents whose productivity is above *w*.
The interpretation of this equation is simple enough. Increasing the marginal tax rate of the agent with level of productivity *w*, the government both wins and loses income. It loses because the agents whose productivity is *w* will decrease their labor supply. The corresponding loss is obtained by multiplying the left side of (3) by the term in *f(w)* on the right – in other words the number of people who are at this level of productivity – and by the term $w/(1+\frac{B''(L)L}{B'(L)})$ – in other words from how much the wage income decrease. The terms staying on the right could be interpreted as the additional income that the government obtains increasing the tax paid in the marginal income bracket corresponding to *w* by all those whose productivity is higher than *w*, that is to say *1-F(w)*. This gain is corrected by the relative difference between the average marginal social value of the corresponding incomes and the average marginal social value of the income of those who effectively pay this supplementary tax.

Equation (3) is the starting point of our analysis. The key ingredients of the optimal tax schedule are the distribution of the productivities *f(w)*, the individual utility function *U(C,L)* and the social welfare function *H(U)*. The objective of our analysis is to use observations on effective marginal tax rate computed by microsimulation models as well as data on gross and disposable incomes of households in order to reveal the government subjective valuation of the individual utility functional form. To achieve this objective it is thus necessary to rewrite equation (3) in a way that can be directly estimated from data. To do it, three main problems have to be solved. Firstly we have to define the shape of social welfare function in a flexible way. Secondly, we have to transform the differential equation on *w* (that, as Bourguignon and Spadaro (2000a) explain, it is not observable directly in data) in a differential equation on *Y* (that is immediately observable in data). Third, we have to rewrite the individual utility function (2) taking into account the agent maximizing behavior implicit in observed gross and disposable income. This implies inverting the individual optimal problem (1.2) and recovering *U(C,L)* starting from *C**, *Y* = wL** and *T(Y*)*.

Concerning the specification of the social welfare function, what has been done in this paper is to use the following functional form:

[5] For the derivation of this equation see Atkinson and Stiglitz (1980) or Atkinson (1995), Diamond (1998), Piketty (1997). At the light of the previous note, this equation could simply be interpreted as a differential equation of the tax function T( ). Its integration gives the redistribution function. The government budget constraint makes it possible to identify the constant of integration T(0) that can be considered as a universal social contract tax (or a transfer if it is negative).

$$H(U) = \frac{(U - U_0)^{\alpha}}{\alpha} \qquad (5)$$

where U_0 is a parameter that control for the social marginal weight of the poorest agent. The parameter α takes values in the interval (-∞, 1]; it defines the concavity of the social welfare function and thus the level of aversion to inequality of the government. If α tends to one the government is Utilitarian; when α tends to -∞ the government become Rawlsian. This function belongs from the class of the Hyperbolic Absolute Risk Aversion (HARA) functions. The reason of our choice is motivated by the higher flexibility that such specification allows in optimal tax calculation: the key parameters controlling for the social weights of each agent are easily identified and modifiable (see Stern 1986).

The second problem concerns mainly the term $\frac{1-F(w)}{w.f(w)}$ in equation (3) (known as the inverse hazard ratio). A way to deal with it is by using the theoretical relation (*Y* = *wL*) among gross labor income, effort and productivity implicit in agent individual utility maximization problem (1.2) and to derive an expression equivalent to the inverse hazard ratio, depending on Y.
In appendix 1 we demonstrate that the following identity holds:

$$\frac{1-F(w)}{wf(w)} = \frac{1-G(y)}{yg(y)}\left[\frac{1+\varepsilon_L(y)\nu(y)\dfrac{t(y)}{1-t(y)}}{1+\varepsilon_L(y)}\right] \qquad (6)$$

where *g(y)* is the distribution density of gross income, *G(y)* is his cumulative, *t(y)* is the effective marginal tax rate on gross income, *t'(y)* his first derivative, $\nu(y)$ is the elasticity of effective marginal tax schedule with respect to gross income [i.e. $\nu(y) = \frac{t'(y)y}{t(y)}$] and $\varepsilon_L(y)$ is the compensated elasticity of labor supply. With the separability restriction imposed to the individual utility function (eq. 2) we have that:

$$\varepsilon_L(y) = \frac{U_L}{LU_{LL}} = \frac{B'(L)}{LB''(L)} \qquad (7)$$

The condition on the inversion of the individual utility maximization problem (the third problem to deal with) is determinate in appendix 2. It gives us the following relation:

$$U(C,L) = C - B(L) = DY - \int_{y\min}^{y}\left(\frac{1-t(x)-\nu(x)t(x)}{1+\dfrac{1}{\varepsilon_L(x)}}\right)dx - \Theta \qquad (8)$$

where *DY* is the observed disposable income (used as proxy of consumption), Θ is a constant.

Using equation (6) and (8) we can rewrite equation (3) obtaining:

$$\frac{t(y)}{1-t(y)}-\left[1+\frac{1}{\varepsilon_L(y)}\right]*\left[\frac{1+\varepsilon_L(y)\nu(y)\dfrac{t(y)}{1-t(y)}}{1+\varepsilon_L(y)}\right]*\left[\frac{1-G(y)}{yg(y)}\right]*$$

$$*\left[1-\frac{\int_{y}^{y\max} H'\left\{DY(z)-\Theta-\int_{y\min}^{y}\left(\frac{1-t(x)-v(x)t(x)}{1+\dfrac{1}{\varepsilon_L(x)}}\right)dx\right\}g(z)dz}{1-G(y)}\right]=0 \qquad (9)$$

that is a non linear equation in $\varepsilon_L(y)$ that can be solved numerically by fixed point algorithms starting from the observation of *t(y), ν(y), g(y), G(y)* and *DY*. This equation is the consolidated form of two inversed optimal problems: the agent utility maximization and social planner utility maximization. The solution of this equation gives us the government subjective valuation on the elasticity of labor supply (and then on the individual utility functions). Its empirical implementation raises some technical problems that will be described in the next section.

2. Basic principles for empirical implementation

a) Continuity and differentiability

The application of the modified optimal taxation formula (9), requires the knowledge of the continuous functions *f(w), t(w)* and their derivatives. Unfortunately, what may be obtained from households data bases is a set of discrete observations of the gross income Y_i, the associated cumulative distribution function, $G(Y_i)$ and, with the availability of a microsimulation model, the marginal tax rate $t(Y_i)$ associated to each observation. The following operations permit to get an estimate of the derivatives of the function g*(w)* and *t(w).*

(i) For any value of Y, we obtain an estimate of the density function g*(Y)* and the effective marginal tax rate *t(Y)* by kernel techniques defined over the whole sample of observations - using a Gaussian kernel with an adaptive window[4]. These kernel approximations are made necessary first by the need to switch from a discrete to a continuous representation of the distribution and the tax schedule and second by the heterogeneity of the population with respect to some characteristics that may influence marginal tax rates and productivity estimates[5].

[4] This choice was justified by the lack of observations and the increasing distance between them in the upper tail of the distribution. For technical details, see Hardle (1990).

[5] Household composition, occupational status and home ownership are examples of sources of heterogeneity with respect to the tax system.

(ii) We estimate the derivatives of *t(y)* using again a kernel approximation computed over the whole sample.[6]

(iii) We compute the elasticity of *t(y)* (i.e. the term $\nu(y) = yt'(y)/t(y)$).

(iv) We solve the non linear equation (9) by fixed-point algorithms, computing $\varepsilon_L(y)$ for different specifications of parameters Θ, U_0 and α.

b) Households with zero income and households with apparently irrational behaviour

In presence of a guaranteed minimum income in a tax-benefit system, some households may find it optimal not to work at all. In the simple labor supply model above, this would correspond to a situation where the marginal tax rate is 100 percent. However, there is some ambiguity about these situations. Practically, some households are observed in pats of their budget constraint where the marginal tax rate is indeed 100 percent. There are two possible reasons for this. First, transitory situations may be observed where households have not yet converged towards their preferred consumption-labor combination. Second, transition periods are allowed by tax-benefit systems where beneficiaries of minimum income schemes may cumulate that transfer and labor income for some time so as to smoothen out the income path on return to activity.

The example of the French minimum income program (RMI) suggests the following way of handling the 100 marginal tax rate issues. People receiving the minimum income RMI and taking up a job lose only 50 percent of additional labor income during a so-called 'intéressement' period – 18 months. At the end of that period, however, they would lose all of it if they wanted to keep benefiting from the RMI. Discounting over time, this means that the actual marginal tax rate on the labor income of an 'RMIste' is between 50 and 100 percent. Taking the middle of that interval, the budget constraint of that person thus writes:

$y = RMI + .25 * wL$ if this person qualifies for the RMI –i.e. $wL < RMI$.

But it is simply: $y = wL$ if $wL > RMI$[7].

This budget constraint is clearly convex. Therefore, there should be a range of labor incomes around the RMI where it would be irrational to be[8]. But, of course, some households are actually observed in that range (this phenomena affects around 1% of the total sample used), which is inconsistent with the model being used and/or the assumption made on the marginal tax rate associated with the RMI. One way of dealing with this inconsistency is to assume that all gross labor incomes are observed with some measurement error drawn from some arbitrary distribution. The measurement error is such that, without it, households would be rational and supply a quantity of labor outside the preceding range. This treatment of the data is analogous to the original econometric model describing the labor supply behavior of households facing a non-linear and possibly discontinuous budget constraint by Hausman (1985).

3. The elasticity of labor supply in practice: what we know?

In econometric estimations of such parameter, there is not agreement about his right value. The results of available studies differ depending on sex, age, marital status, number of children, occupational status, level of income and estimation procedures. A recent survey of estimation techniques and results (overall for UK and US studies) by Blundell and MaCurdy (1999) gives a range of values mostly concentrated in the interval (0, 1). For France,

[6] For technical details about the computations of kernel derivatives see Pagan and Ullah (1999, pag. 164).

[7] All other benefits that may complement the RMI are ignored in this argument, but they are taken into account in the calculations made below.

[8] This interval may easily be computed using the preference function of households and the budget constraint described by the preceding conditional system. Note that it depends on the size and the socio-demographics characteristics of each household.

Bourguignon and Magnac (1991), Donni (2000), Bargain (2004), Choné et al. (2003) and Laroque and Salaníe (2002) find labour suplly estimates in the same interval. In particular, 0.1 is around the average elasticity of men and 0.5 around the average elasticity of married women [if they have children, the values are higher (see Bargain 2004)]. Very similar results are obtained in the case of Spain (see Martinez-Granado 2002 and Segura-Bonet 2002).
Similar results have been obtained on the basis of the relationship between taxable incomes and changes in tax rates. The difference in difference estimation performed on tax returns panel data by Feldstein (1995) using the 1986 US tax reform as a natural experiment yields estimates on the high side, in the interval [1, 3]. However, more recent work by Auten and Carroll(1999) and Gruber and Saez (2002) have questioned such high values. In particular, they have shown that Feldstein's estimates were probably affected by a bias coming from the mean reversion tendency present in panel income data. Improving on Feldstein's methodology lowered the estimates of the elasticity of US taxable income to values around 0.7 in Auten and Carroll (1999), and 0.4 in Gruber and Saez (2002). In the case of France, Piketty (1998) found elasticies of taxable income even lower, around 0.2. For Spain, Diaz-Mendoza and Gonzalez-Páramo (2004) find elasticites around 0.35.

4. Microsimulation models, datasets and data treatment.

The methodology, which has just been presented, has been applied to data from France and Spain.
For France, the samples and the micro simulation model were taken from EUROMOD, a project whose objective is to build an integrated micro simulation model for the 15 countries of the European Community. A complete and detailed description of the EUROMOD micro simulation model as well as the datasets is contained in Sutherland (2001). The version of the model used in this paper replicates the laws enforced in 1995 in France. All the modules replicate social contributions levied on wages (for employers and employees) and on self-employed workers; social contributions on other types of income (unemployment benefits, income from pensions and capital return); income taxes; family benefits and social assistance mechanisms. The datasets used for France are the 1995 Households Budget Survey of INSEE.
For Spain, we use is the 1995 Spanish database from the European Community Household Panel (ECHP), published by EUROSTAT, since it includes socio-demographic characteristics, income characteristics and labor status. Our dataset contains information at both individual and household levels. The original dataset was then updated, using a correction factor including inflation and the growth rate from 1994 to 1999 (the updating factor used has been 1.335). No changes in the socio-demographic structure were taken into account. The micro-simulation model, called GLADHISPANIA, replicates the legislation in force in 1999 on social contributions levied on wages and on self-employed workers; social contributions on other types of income (unemployment benefits, income from pensions and capital return) and income taxes. A complete and detailed description of the GLADHISPANIA micro simulation model as well as the datasets is contained in Oliver and Spadaro (2004).
To keep with the logic of the optimal (labor income) taxation model, all households with zero income and with non-labor income, including pension and unemployment benefits above 10 per cent of total income, were eliminated from the samples. The final samples used in the paper contain, in the case of France, 5527 households (on a total of 10214), 963 of which are singles. In the Spanish case, the final sample size is 2.718 (on a total of 6420); 326 of which are singles[9].

[9] Off course the final samples used cannot be considered as representative of the whole population. They are instead representative of the workers population.

The microsimulation models have been used in order to compute the effective marginal tax rate for each household. This variable was obviously not present in the surveys and it was therefore necessary to compute it. The definition of effective marginal tax rate used was the derivative, in each point, of the budget constraint. A possible method of calculation consists of the assignment of a lump-sum amount of gross income to each household (in our case the equivalent of 800 Euro per household per year) and, in the computation with the micro-simulation model, of a new distribution of disposable incomes. The effective marginal rate of taxation is thus obtained from the formula:

$$emtr = \frac{\Delta Taxes + \Delta Benefits}{\Delta Gross\ \mathrm{I}ncome} = 1 - \frac{\Delta Yd}{\Delta y} \qquad (11)$$

5. Results.

Following the last set of remarks in the preceding sections, several applications have been run. They differ with respect to the value selected for the parameters Θ, U_0 , α and the choices made for handling household size.

Practically, actual tax-benefit systems discriminate households according to various characteristics. Size and household composition are the main dimensions along which this discrimination is taking place. The issue thus arises of the way in which these characteristics can be implicitly or explicitly incorporated in the imputation of the social welfare function.

In the results shown below, the size of households is simply ignored. We treat each household as a unique agent. The implications of this choice are somewhat ambiguous. On the one hand, a larger family - in terms of the number of potentially active adults - will generally have a greater gross labour income. On the other hand, it will also face a different marginal tax rate. If the marginal tax rate is a decreasing function of household size for a given household income, as in most tax-benefit systems, then the preceding bias will be attenuated.

We show also the results for the sub-sample of singles (i.e. one person living alone). The view behind this second set of results consists of considering groups of households with the same size or the same composition as populations, which the redistribution authority seeks to maximize social welfare independently of each other. In other words, the optimal taxation problem involves finding an optimal tax-benefit schedule *separately for each household group*. This is implicitly done under some exogenous budget constraint, which makes the aggregate redistribution of income across the various groups of households exogenous. Looking only at singles allows minimizing any possible distortion due to the use of a one-dimensional optimal income tax model.

In figure 1 we show the main input of our analysis, i.e. the effective marginal tax rates in France in 1995 and in Spain in 1999, as computed with the microsimulation model using equation (10), ordered by centiles of gross income. Observing the figure we see that, in both countries, there is an important heterogeneity in the way the systems treat households with the same income. This is certainly due to the fact that, marginal tax rates depend on income but also on other household characteristics. The black lines are the adaptive kernel interpolations of the effective marginal tax rates computed for each household with the microsimulation model.

In the case of France, the original marginal tax rate curve has a U-shape. It is extremely high at the bottom of the distribution because of households facing high marginal tax rates due to the minimum guarantee (RMI). Then, the marginal tax rate falls until a little after the median and then increases slowly with the progressivity of the income tax. In the Spanish case, where no minimum income guarantee does exist, the effective marginal tax rate start from 20% and increases regularly reaching, at the end of the distribution, values around 50%.

The results obtained are presented in figures 3,4, 5 and 6. Figure 3 and 5 shows the revealed elasticity computed on the whole sample (respectively for France and Spain) while figure 4 and 6 show the same results for a sub sample of singles. In each figure there are two panels. Each of it contains the results obtained under some hypothesis on the level of minimum individual welfare that government want to guarantee to each individual (i.e. the terms U_0 in equation 5) and for some given level of aversion to inequality (i.e the parameter α in equation 5) calibrated in order to get an average labor supply elasticity of 0.1 (red lines) and 0.5 (black lines). In each panel we show the value of the parameters retained. It is important to remark that as α tends to one ($-\infty$) the government becomes more utilitarian (rawlsian). It is also important to remark that, given the specification of the social welfare function retained, the value of $\Theta+U_0$ is determinant for the social marginal weight of each agent. It must be included in the interval $(-\infty, 0]$ given that, for positive values, the social welfare function of the poorest agent is not defined. In particular, if $\Theta+U_0 = 0$, the marginal social weight of the poorest agent is $+\infty$, if $\Theta+U_0 < 0$ then the weight of the poorest agent is positive but not infinite[10].
The following table resumes the values of each parameter we present in figures 3, 4, 5 and 6.

Table 1. Definition of different scenarios simulated and presented.

France			
$\Theta = 0$	Average $\varepsilon_L(y)$	***Singles***	***All Households***
$U_0 = 0$	0.1	$\alpha = 0.74$	$\alpha = 0.68$
	0.5	$\alpha = 0.09$	$\alpha = -0.01$
$U_0 = -15.000$	0.1	$\alpha = 0.68$	$\alpha = 0.65$
	0.5	$\alpha = -0.16$	$\alpha = -0.14$
Spain			
$\Theta = 0$	Average $\varepsilon_L(y)$	***Singles***	***All Households***
$U_0 = 0$	0.1	$\alpha = 0.995$	$\alpha = 0.962$
	0.5	$\alpha = 0.939$	$\alpha = 0.876$
$U_0 = -15.000$	0.1	$\alpha = 0.99$	$\alpha = 0.95$
	0.5	$\alpha = 0.91$	$\alpha = 0.84$

The results show that there are some common features in the two systems analyzed.
The first feature is that for a given redistribution system, if the government is supposed to be more inequality averse, the elasticity of labor supply implicit in the optimal income tax problem is higher. Take for example the case of France, all household, $U_0 = 0$: in this scenario we find that with a "rawlsian" specification of the aversion to inequality ($\alpha=-0.01$) we obtain an average elasticity five times higher (0.5 instead of 0.1) than with an "utilitarian" specification as $\alpha=0.68$.
The intuition is the following. The observed marginal tax schedule, supposed to be the optimal one, is the best solution of the trade off between equity and efficiency concerns. If two governments, with different inequality aversion, solve the optimal tax problem in the same way, it means that they assign different weight to the efficiency problems. In particular,

[10] To see it, we must insert in the first derivative of (5) with respect to U, the equation (8). We will observe that $\Theta + U_0$ determine the position of the vertical asymptote and, then, the weight of the poorest agent.

the more inequality averse is giving more importance to efficiency problems than the less averse; otherwise the optimal marginal tax rate would have been higher.
Interestingly, when performing the same type of simulation in the Spanish case, we find that (left panel of figure 5) to obtain an average elasticity of 0.5 instead of 0.1 we only need to switch from $\alpha=0.962$ to $\alpha=0.876$. This strong asymmetry is present in all the scenarios simulated. It certainly depends on the differences in the shape of the marginal tax rates (see figure 1) but also on the difference in the gross income distributions.

The second, and more important, feature is that the implicit government beliefs about the value of labor supply elasticity have an inverted U-shape. In all scenarios, it seems that both French and Spanish governments assign low values to the labor supply reactions of poor and rich and high values to the elasticity of the middle class.
In the case of France, the maximum value is assigned to people around centile 10 (the elasticity takes values around 0.8 in the case of high aversion to inequality and 0.18 in the other one). On the contrary, the Spanish figures show that the peak of the "inverted U" is much more on the left (i.e the around the fourth-fifth centiles).

In the case of France, the intuition is straightforward: government wants to redistribute income by using the labor income tax as instruments knowing that efficiency problems related to redistribution are higher when individual labor supply is highly sensitive to changes in net wage. If the observed effective marginal tax rate has an U-shape (it is the case for France) it follows that, ceteris paribus, efficiency problems are more important in the middle class range of incomes.
In the case of Spain, on the contrary, the results clearly show that the marginal tax rate is not the only important determinant of our inversion. The Spanish marginal tax rate has not an U-shape. It implies that the preceding "French" argument, alone, doesn't hold. The reason is that the income density distribution plays an important role -as we can see from equation (9). To show it, we have plotted in figure 2 the income distributions both for singles and for the whole samples, normalizing the incomes in order to have the same average in the two countries. We can observe that the Spanish distributions are more dispersed around the average. The fact that, in Spain, we observe more people at the tails of the distribution implies that the disincentive effects of an increase in the marginal tax rates in such portions of the distribution will be higher than in France (see the optimal tax formula 3). It explains, jointly with the shape of the marginal tax rate, the differences in the elasticities we have obtained in our simulations.
Given the complexity of equation (9) it is impossible to characterize analytically the dependence of the elasticity revealed with the inversion of the tax problem from the marginal tax rate and the shape of the income density. For this reason we have performed several numerical simulations consisting in replacing the marginal tax rate of one country on the income distribution of the other. From this exercise we have observed that the shape of the marginal tax rate influence the level of the average elasticity computed but do not have any impact on his shape. On the contrary, it seems that the relevant factor, determining the inverted U-shape of the elasticity curve, is the income density. In figure 7, we show the results of the symmetric exercise implemented on singles[11]. On the left panel we show the results of imposing the French marginal tax rate on the Spanish income distribution, specifying the social welfare function in the same way of what has been done in figure 6 ($\Theta+U_0$ = -15000; α =0.91). We can observe that the shape of the new curve (red line) is very

[11] We have performed simulations both on singles and all households and for several specifications of the social welfare function. The results are very similar.

similar to the initial one except for the level: the average elasticity computed in now 0.2 (against 0.5 of the initial situation). Similarly, (right panel) when performing the symmetric exercise (i.e. the Spanish marginal tax rate on the French income distribution with the following social welfare function parameters: $\Theta+U_0$ = -15000; α =-0.16) we observe that the new elasticity curve (red line) is shifted upward until an average value of 1.25 keeping a very similar shape with respect to the initial one.

Another important common feature concerns the difference among the results for households treated as unit of analysis and the results for singles. Even if the shape of the elasticity curves do not changes, we observe that the revealed value of elasticity in the case of singles is slightly different than for the case of the whole sample. In particular we find that it is higher in the case of France and lower in the case of Spain. This result certainly depends on the way the two governments weight the family size and control for the redistribution among different types of households. Unfortunately, in our (one-dimensional) optimal taxation framework it is impossible to say more about that.

The robustness of the preceding results has been tested by performing sensitivity analysis to changes in the parameters of the social welfare function (Θ, U_0 and α). Changes in $\Theta+U_0$ produce a very small quantitative impact on the shape of the elasticity estimated (as one can easily observe). The results are more sensitive with respect the parameter α: the higher is α the lower is the average elasticity the flatter is the curve.
In order to test robustness of the results with respect to the statistical representativity of our samples and to the numerical analysis procedure, we have performed a bootstrapping analysis consisting in replicating all the numerical computations on 1000 alternative sub samples generated randomly from the empirical distribution of the original one[12]. The results are extremely satisfactory: lower and upper band of confidence intervals (at 95%) are very similar to the estimated curve (for this reason we do not show it in the figures).

6. Conclusions

This paper has explored an original side of applied optimal taxation. Instead of deriving the optimal marginal tax rate curve associated with some distribution of individual productivities, the analysis consists of retrieving the government beliefs about the individual preferences that makes the observed marginal tax rates optimal under an arbitrary assumption about the social welfare function of the government. . We restrained the analysis to the class of separable and quasi-linear (in consumption) utility functions. In this case, there is an immediate relationship between the form of the utility function and the elasticity of labour supply (given by the equation 7). This relation has been exploited in the optimal tax calculations.
The results show that, in the two countries analyzed, the government's beliefs about the value of elasticity all along the income range are not uniform. Poor and rich people react less than the middle class. This result underscores the limitations of using estimates of average labour supply elasticity to analyze fiscal reforms.
We have also shown that the heterogeneity of the treatment of household size, implicit in the tax system analyzed, influences dramatically the elasticity calculated. This highlights the relevance of defining a theoretical framework allowing for the explicit treatment of multidimensional optimal redistribution.
Another important lesson is the practical interest of the results. It is customary to discuss and evaluate reforms in tax-benefit systems in terms of how they would affect some 'typical

[12] See Appendix 3.

households' and more rarely what their implications are for the whole distribution of disposable income. The instrument developed in this paper offers another interesting perspective. By drawing implicit elasticity curves consistent with a tax-benefit system before and after reforms, it is possible to characterize in a more precise way the efficiency and also the distributional bias of a reform.

APPENDIX 1: Analytical derivation of the identity (6).

In this section we demonstrate the following relationship:

$$\frac{1-F(w)}{wf(w)} = \frac{1-G(y)}{yg(y)}\left[\frac{1+\varepsilon_L(y)\nu(y)\dfrac{t(y)}{1-t(y)}}{1+\varepsilon_L(y)}\right] \qquad (6).$$

We start rewriting $y = wL$ as:

$$y = \varphi(w) \Rightarrow w = \varphi^{-1}(y) \qquad (11)$$

We can then replace (11) in *f(w)* and *F(w)* obtaining:

$$F(w) = G[\varphi(w)] \Rightarrow f(w) = \frac{dG[\varphi(w)]}{dw}\frac{d\varphi(w)}{dw} = g[\varphi(w)]\varphi'(w) \qquad (12).$$

The inverse hazard ratio can now be written as:

$$\frac{1-F(w)}{w.f(w)} = \frac{1-G(y)}{w.g(y)\varphi'(w)} \qquad (13)$$

The problem now is to find a useful expression for $\varphi'(w)$. We can start by differentiating $\varphi(w)$ obtaining:

$$\varphi'(w) = \frac{d\varphi(w)}{dw} = \frac{dy}{dw} = \frac{Ldw + wdL}{dw} = L + w\frac{dL}{dw} \qquad (14).$$

The term $w\dfrac{dL}{dw}$ can be obtained manipulating the first order condition of the agent utility maximization problem (1.2) as follows:

Differentiating the f.o.c. on *w* and on *L* $w = \dfrac{b(L)}{1-t(wL)}$ we obtain (after some rearrangement):

$$\frac{dL}{dw} = \frac{1-\dfrac{bLt'}{(1-t)^2}}{\dfrac{b'}{1-t}+\dfrac{bwt'}{(1-t)^2}} \qquad (15)$$

Multiplying both sides for *w/L* and performing a little bit of tedious algebra we obtain:

$$w\frac{dL}{dw} = L\frac{\varepsilon_L - \dfrac{\varepsilon_L * v * t}{1-t}}{1+\dfrac{\varepsilon_L * v * t}{1-t}} \qquad (16)$$

(remember that: $\varepsilon_L(y) = \varepsilon_L = \frac{U_L}{LU_{LL}} = \frac{B_L}{LB_{LL}} = \frac{b}{Lb'}$ and that $\nu(y) = v = \frac{t'(y)y}{t(y)}$)

Replacing (16) in (14) and multiplying both sides for w give us:

$$w\varphi'(w) = y * \frac{1+\varepsilon_L(y)}{1+\varepsilon_L(y)\nu(y)\frac{t(y)}{1-t(y)}} \qquad (17)$$

Replacing (17) in the right side of equation (13) give us the equation (6).

APPENDIX 2: Analytical derivation of the identity (8)

In this section we demonstrate the following relationship:

$$U(C,L) = C - B(L) = DY - \int_{y\min}^{y} \left[\frac{1-t(z)-\nu(z)t(z)}{1+\frac{1}{\varepsilon_L(x)}} \right] dx - \Theta \qquad (8)$$

The first order condition of the agent utility maximization problem (1.2) gives us the following relation:

$$w = \frac{b(L)}{1-t(wL)} \qquad (22) \qquad \text{where} \qquad b(L) = \frac{dB(L)}{dL} \qquad (23)$$

Equation (22) can be used to rewrite the definition of gross labour income (y = wL) as follows:

$$y = L\frac{b(L)}{1-t(wL)} \qquad (24)$$

Differentiating (24) on w and L we get:

$$dy = \left[\frac{b'L}{1-t} + \frac{b}{1-t} + \frac{bLwt'}{(1-t)^2} \right] dL + \frac{bL^2t'}{(1-t)^2} dw \qquad (25).$$

Using the fact that $y = wL \Rightarrow dy = wdL + Ldw \Rightarrow dw = (dy - wdL)/L$ we can rewrite (25) in the following way:

$$dy = \left[\frac{b'L}{1-t} + \frac{b}{1-t} + \frac{bLwt'}{(1-t)^2} \right] dL + \frac{bLt'}{(1-t)^2} dy - \frac{bLt'}{(1-t)^2} wdL \qquad (26)$$

that after some rearrangement become:

$$dy\left[1 - \frac{bLt'}{(1-t)^2} \right] = \left[\frac{b'L}{1-t} + \frac{b}{1-t} \right] dL\ . \qquad (27)$$

Performing some algebra we obtain:

$$\frac{dy}{bdL} = \frac{1+1/\varepsilon_L(y)}{1-t(y)-\nu(y)t(y)} \Rightarrow dB = \frac{1-t(y)-\nu(y)t(y)}{1+1/\varepsilon_L(y)} dy \qquad (28)$$

with $\varepsilon_L(y) = \frac{U_L}{LU_{LL}} = \frac{B_L}{LB_{LL}} = \frac{b}{Lb'}$ and $\nu(y) = \frac{t'(y)y}{t(y)}$.

Integrating both sides on y give us:

$$B(L) = \theta + \int_0^y \frac{1 - t(z) - \nu(z)t(z)}{1 + 1/\varepsilon_L(z)} dz \qquad (29)$$

and finally (using disposable income *DY* as proxy of consumption).

$$U(C, L) = C - B(L) = DY - \theta - \int_0^y \frac{1 - t(z) - \nu(z)t(z)}{1 + 1/\varepsilon_L(z)} dz \qquad (8)$$

where θ is a constant of integration.

Appendix 3: The bootstrap principle [based on Bradley and Tibshirani (1993)].

The problem solved by bootstrapping can be formulated as follows. We have a random sample $X = (x_1, \ldots x_n)$, obtained from an unknown probability distribution A and we want to estimate a parameter (e.g. the average change in labor supply) $\theta = t(A)$ on the basis of *X*. We calculate an estimation of $\hat{\theta} = s(X)$ using *X;* then the problem is to know how accurate this estimate is.

Bootstrapping technique is based on re-sampling with replacement. Each bootstrap sample *X** is an independent random sample of size *n* from the empirical distribution followed by *X* (that we call $\hat{A}$). To each bootstrap sample it corresponds a bootstrap estimation of $\hat{\theta}$: $\hat{\theta}^* = s(X^*)$ that is the results of applying to *X** the same function *s( )* which has been applied to *X*. The bootstrap algorithm for estimating the standard error and the confidence intervals can be summarized by the following four steps:

- Select *B* independent bootstrap samples $X_1^*, X_2^*, \ldots, X_B^*$ each consisting of *n* data values drawn with replacement from *X* (a good rule of thumb is B = 1000).
- Evaluate the bootstrap replication corresponding to each bootstrap sample $\hat{\theta}^*(b) = s(X_b^*)$ with $b = 1, 2, \ldots, B$
- Estimate the standard error using the formula:

$$s\hat{e}_B = \left\{ \sum_{b=1}^{B} \left[\hat{\theta}^*(b) - \sum_{b=1}^{B} \hat{\theta}^*(b) / B \right]^2 / (B-1) \right\}^{1/2}$$

- And the confidence intervals as: $\left[\hat{\theta} - z^{(1-\alpha)} s\hat{e}_B;\ \hat{\theta} + z^{(\alpha)} s\hat{e}_B\right]$ where z^{α} is the α^{th} percentile of the standardized normal distribution.

References

Atkinson A.B, (1995) "Public Economics in Action: Basic Income-Flat Tax Proposal" Clarendon Press Oxford.

Atkinson A, Stiglitz J. (1980), "*Lectures on Public Economics*" McGraw Hill International Editions.

Auten, G. and Carroll, R. (1999) "The Effect of Income Taxes on Household Income", *Review of Economic and Statistic* 81, 681-693.

Bargain O., (2004) "*On Modelling Household Labour Supply With Taxation*", DELTA mimeo.

Blundell, R., MaCurdy (1999), "Labour Supply: a Review of Alternative Approaches", *Handbook of Labour Economics vol 3a*. North Holland.

Bourguignon F., Magnac T., (1990), "Labour Supply and Taxation in France", *Journal of Human Resources*, vol 25, n.3.

Bourguignon, F., Spadaro A. (2000) "Redistribution et incitations au travail: une application simple de la théorie de la fiscalité optimale"; *Revue Economique* nº3 vol 51.

Bourguignon, F., Spadaro A.-(2000b), "Social Preferences Revealed through Effective Marginal Tax Rates". *DELTA Working Paper* nº 2000-29.

Bourguignon F, Spadaro A., (2002) "*Tax-Benefit Revealed Social Preferences: Are Tax Authorities Non-Paretian?*" mimeo

Bradley, E and J. Tibshirani, (1993) "*An Introduction to the Bootstrap*" Chapman & Hall eds, New York.

Choné, P., Laroque G., (2001), "Optimal Incentives for Labour Force Participation", *CREST Working Paper* nº 2001-25, INSEE Paris

Choné, P., Le Blanc D., Robert-Bobée I., (2003), "Female labour Suplly and Child Care in France", *CREST Working Paper* nº 2003-4, INSEE Paris

D'Autume, A. (2001), "L'imposition optimale du revenu: une application au cas français" *Revue Française d'Economie*, 3, pp.3-63

Diamond, P. (1998), "Optimal Income Taxation: An Example with U-Shaped Pattern of Optimal Marginal Tax Rate", *American Economic Review*, vol 88, n.1.

Diaz-Mendoza, M. and Gonzalez-Páramo, J.M. (2004), "*La respuesta de los contribuyentes ante las reformas del IRPF: Elasticidad de la base imponible y tarifas óptimas*" Mimeo.

Donni, O. (2000), "*Essais sur les modèles collectifs de comportement du menage*" Ph.D. Dissertation EHESS, Paris.

Feldstein, M. (1995), "The Effect of Marginal Tax Rates on Taxable Income: A Panel Study of the 1986 Tax Reform Act", *Journal of Political Economy*, 103, pp. 551-572.

Gruber, J. and Saez, E. (2002), "The Elasticity of Taxable Income: Evidence and Implications", *Journal of Public Economics*, 84, pp. 1-32.

Härdle W., (1990) "*Applied nonparametric regression*" Cambridge University Press.

Hausman, J. (1985), "The Econometrics of Nonlinear Budget Set", *Econometrica*, vol 53, pp.1255-82.

Laroque G., Salanié B., (2002) "*Institutions et emploi, le marché du travail des femmes en France*" Economica.

Kurz M. (1968) "On the Inverse Optimal Problem," in *Mathematical Systems Theory and Economics*, edited by H. W. Kuhn and G. P. Szego, Springer-Verlag (1968); 189-202.

Martínez-Granado M. (2002), "Oferta de trabajo femenina en España: un modelo empírico aplicado a mujeres casadas", *Cuadernos de ICE*, 67, pp. 130-152.

Mirrlees J.A, (1971), "An Exploration in the Theory of Optimum Income Taxation", *Review of Economic Studies*, n.39.

Mirrlees J.A, (1986), “The Theory of Optimal Taxation”, in *Handbook of Mathematical Economics*, vol. III, Arrow and Intriligator eds, North Holland, Amsterdam.

Oliver X., Spadaro, A., (2004), “A Technical Description of GLADHISPANIA: A Spanish Micro-Simulation Tax-Benefit Model” *DEA WP* nº7-2004. Universitat de les Illes Balears, http://www.uib.es/depart/deaweb/deawp/

Pagan A., Ullah A., (1999) « *Nonparametric Econometrics* » Cambridge University Press.

Piketty T., (1997), « La redistribution fiscale contre le chômage », *Revue Française d’Economie*, vol. 12n n° 1, pp. 157-203.

Piketty T., (1998), « L’impact des incitations financières au travail sur les comportements individuels : un estimation pour le cas français», *Economie et Prévision*, n° 132-133 pp. 1-35.

Saez, E. (2001), Using Elasticities to Derive Optimal Income Tax Rates, *Review of Economic Studies* vol.68, pp. 205-229

Salanié, B. (1998), “Note sur la Taxation Optimale”, *Rapport au Conseil d'Analyse Economique*, La Documentation Française, Paris

Segura-Bonet M. (2002), “Comportamiento (condicionado) de las parejas casadas en materia laboral. Una evidencia para el caso español”, *Cuadernos de ICE,* 67, pp. 153-181.

Stern, N. (1986), “On the Specification of Labour Supply Functions”, in Blundell, R. and Walker, I. (eds.), *Unemployment, Search and Labour Supply*, Cambridge University Press.

Sutherland H., (2001) "Final Report EUROMOD: An Integrated European Benefit-Tax Model”. EUROMOD Working Paper nº EM9/01.

Tuomala, M. (1990), “Optimal Income Tax and Redistribution”, Oxford University Press.

Figure 1. Effective marginal tax rate for France in 1995 and for Spain in 1999.

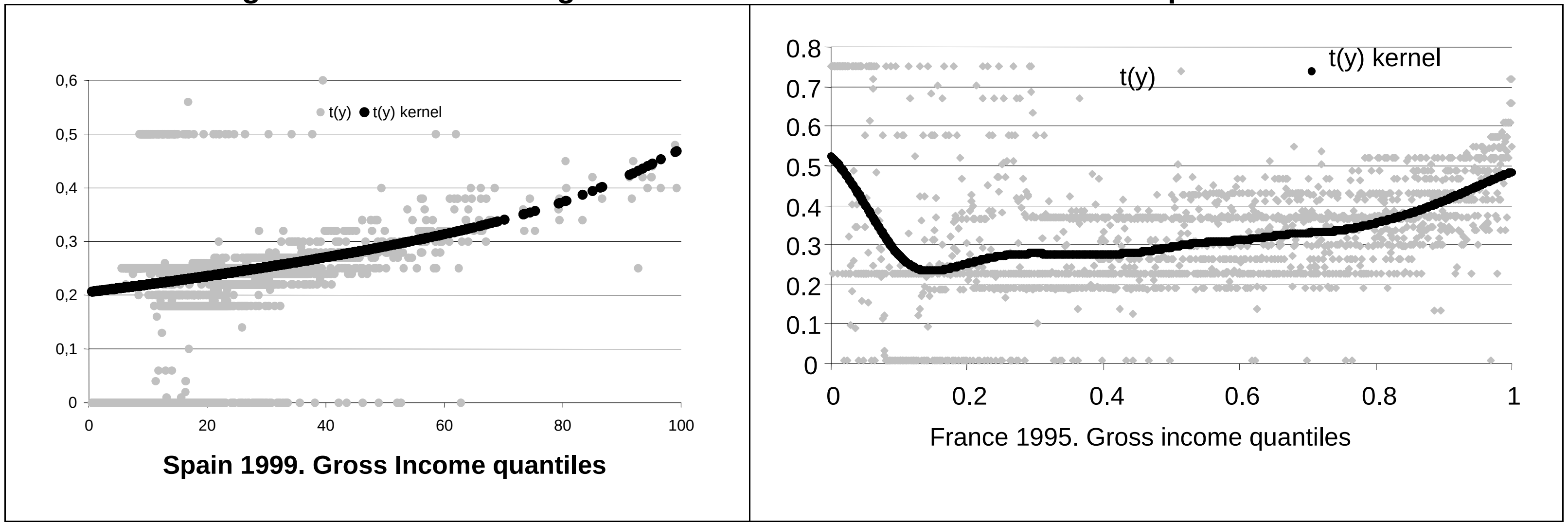


Figure 2. Gross Income Density Distribution in France and Spain. All Households and Singles.

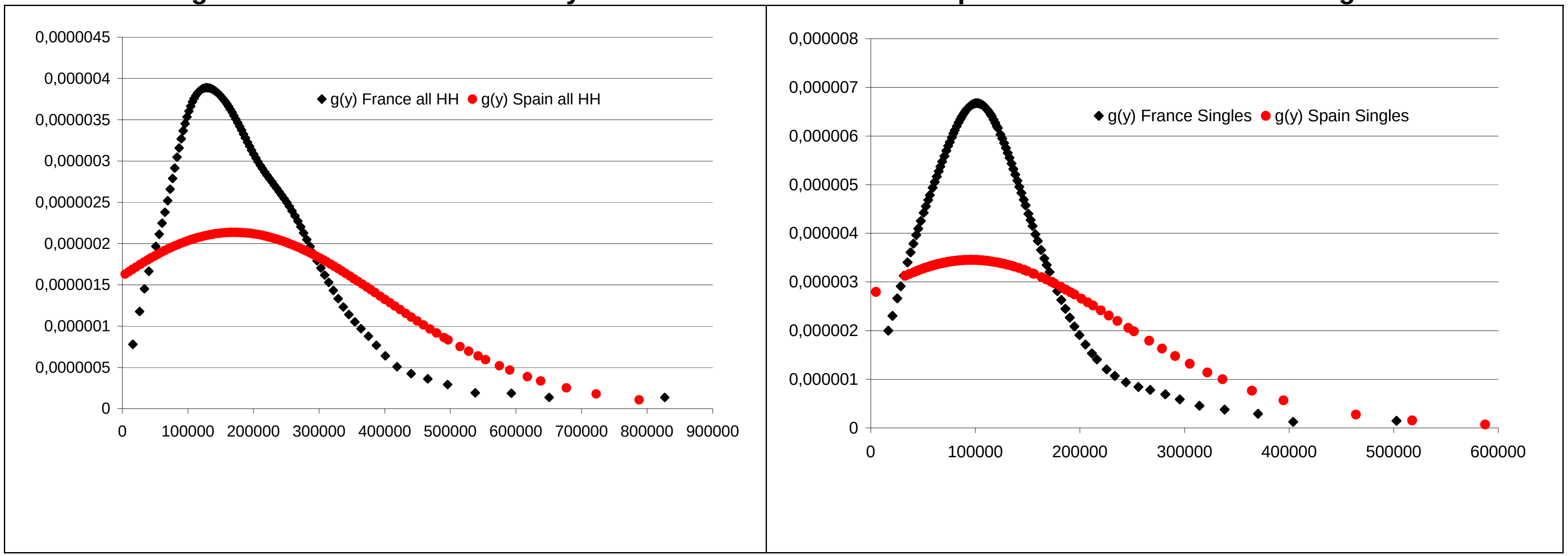
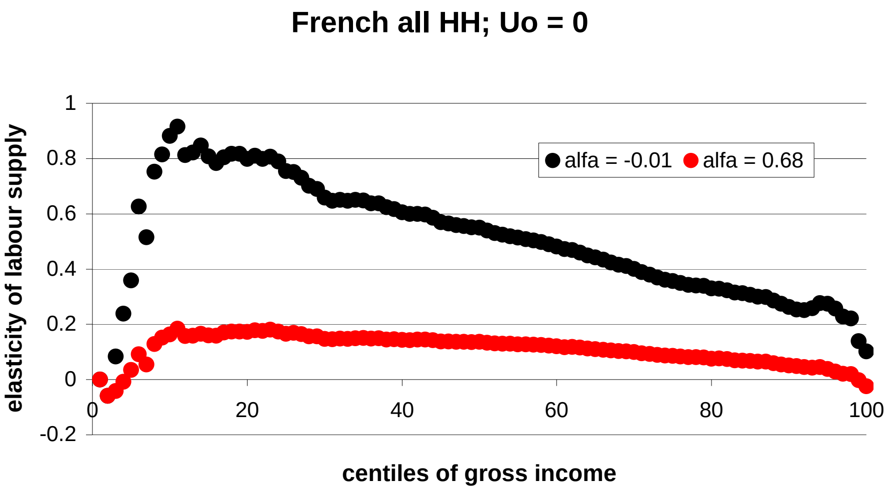


Figure 3. Revealed Elasticity: France, All Households



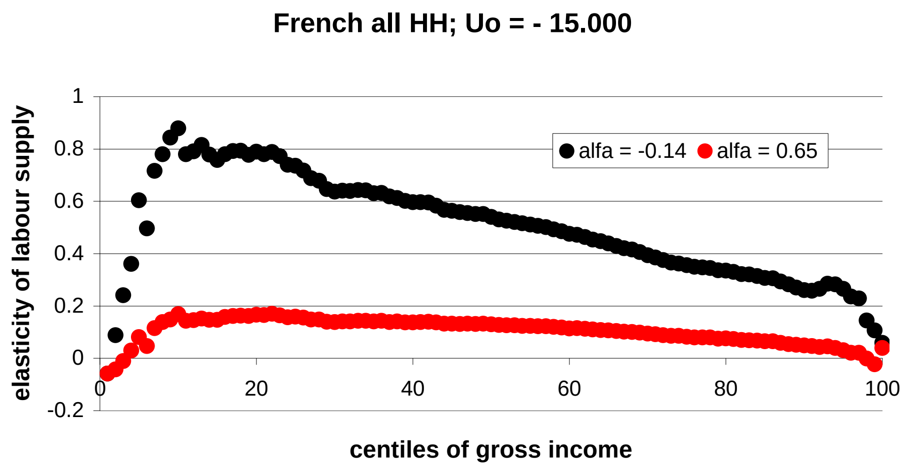
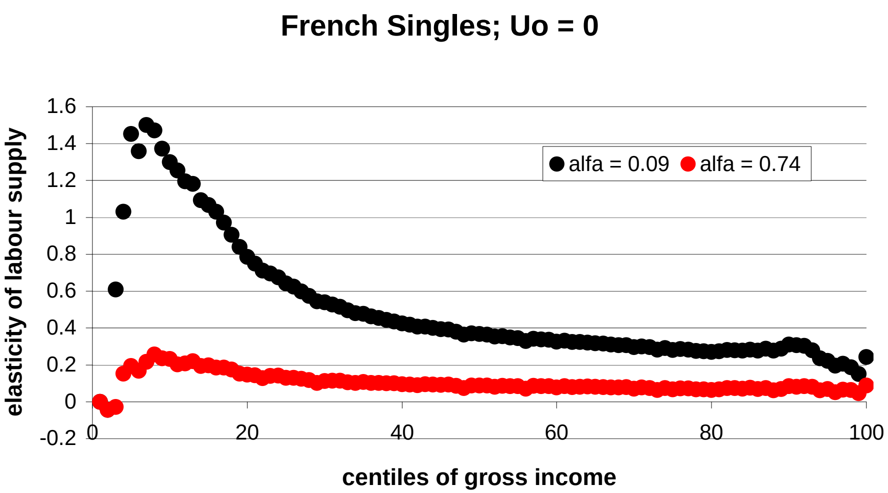


Figure 4. Revealed Elasticity: France, Singles



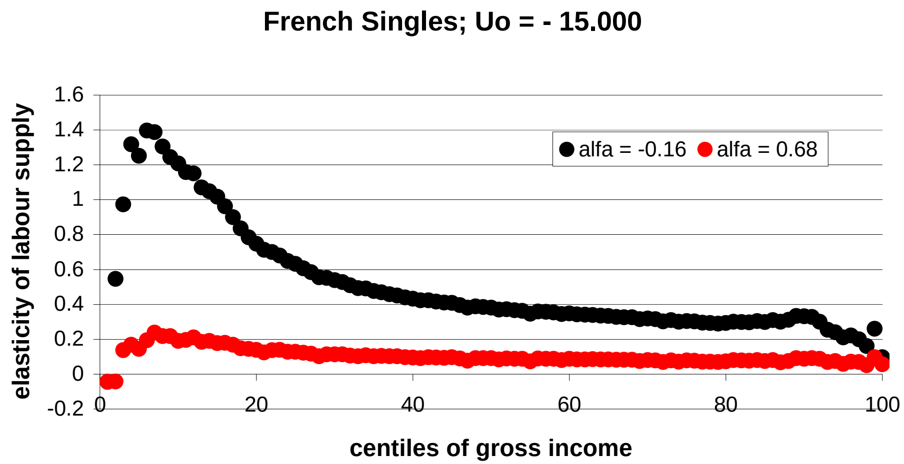


Figure 5. Revealed Elasticity: Spain, All Households

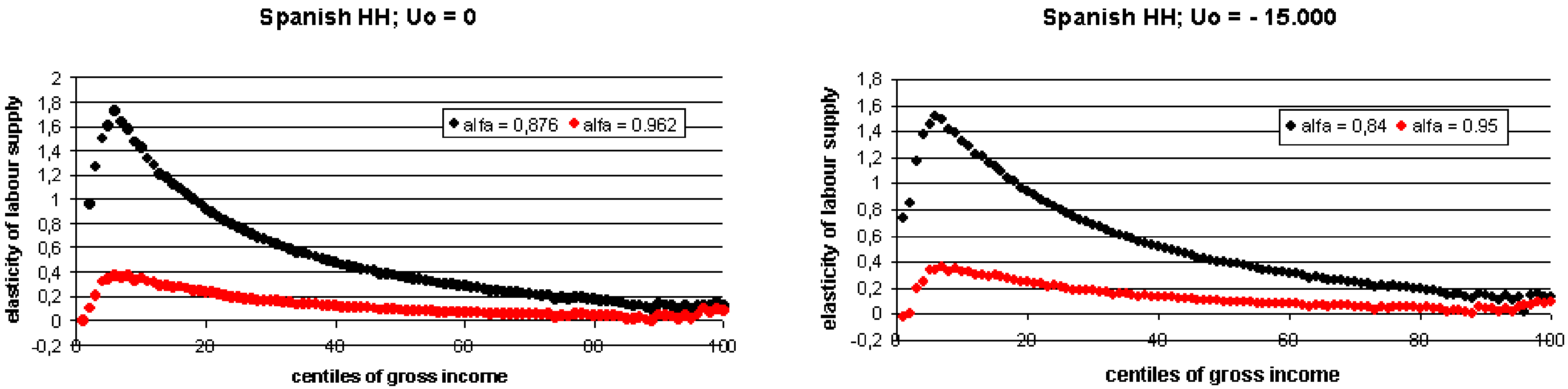


Figure 6. Revealed Elasticity: Spain, Singles

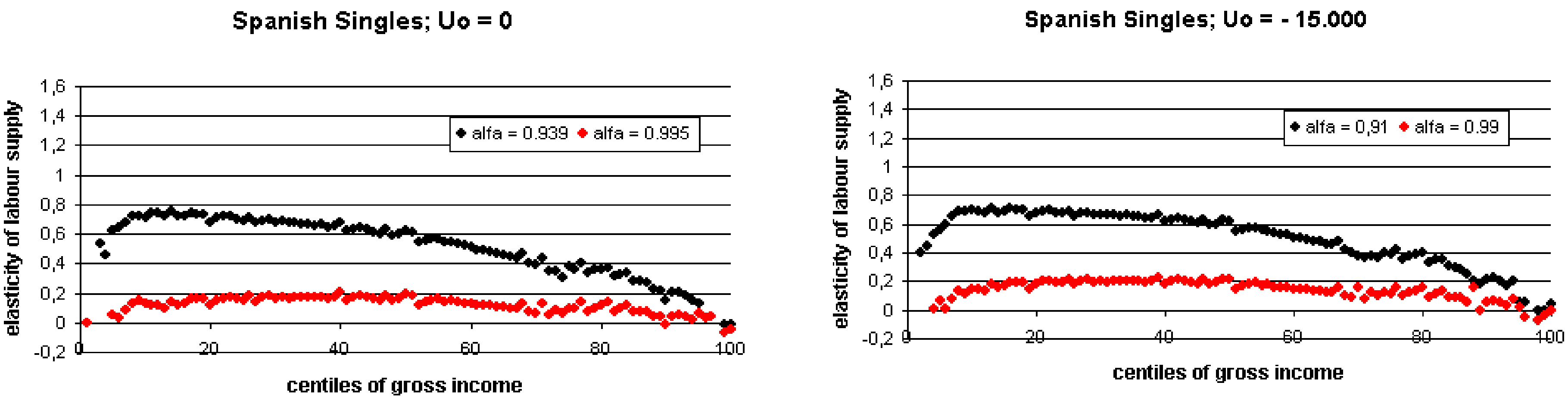


Figure 7. Symmetric experiments: French tax system on Spanish income distribution (left panel) and vice versa (right panel).