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|----------------|-------------------------------------|
| Academic year  | 2015-16                             |
| Subject        | 11001 - Dynamical Systems and Chaos |
| Group          | Group 1, 1S                         |
| Teaching guide | D                                   |
| Language       | English                             |

## Teaching guide

### Subject identification

|                          |  |
|--------------------------|--|
| <b>Subject</b>           | 11001 - Dynamical Systems and Chaos  |
| <b>Credits</b>           | 1.5 de presencials (37.5 hours) 4.5 de no presencials (112.5 hours) 6 de totals (150 hours). |
| <b>Group</b>             | Group 1, 1S  |
| <b>Teaching period</b>   | First semester   |
| <b>Teaching language</b> | English  |

### Professors

| Lecturers                      | Horari d'atenció als alumnes |                |     |            |             |   |
|--------------------------------|------------------------------|----------------|-----|------------|-------------|---|
|                                | Starting time                | Finishing time | Day | Start date | Finish date | Office  |
| Damià Agustí Gomila Villalonga |                              |                |     |            |             | You need to book a date with the professor in order to attend a tutorial. |
| Manuel Alberto Matias Muriel   |                              |                |     |            |             | You need to book a date with the professor in order to attend a tutorial. |

### Contextualisation

#### INSTRUCTORS

Manuel Matias holds a PhD in Physics (1997) and has five 6-year terms ("sexenios") recognized by the ANEP. He has research experience in several aspects of Nonlinear Dynamics, like Chaos Theory, coupled oscillators, synchronization, dynamics of localized structures in extended media, etc.

Damia Gomila holds a PhD in Physics (2003) and has two 6-year terms ("sexenios") recognized by ANEP. He has research experience in the Nonlinear Dynamics of extended media and Nonlinear Optics.

#### COURSE

This is one of the compulsory courses of the Structural Module of the master of Physics of Complex Systems. It is intended to provide a solid background on dynamical systems which will be needed for the other courses of the master.

### Requirements

#### Recommendable

It is recommended that the student has a basic knowledge on differential equations and numerical integration of differential equations (Euler and Runge-Kutta methods).

### Skills

This course develops both specific and generic competences.



## Teaching guide

### Specific

- \* E8: To know to characterize generic behavior of dynamical systems and their instabilities..
- \* E9: To know stability analysis techniques and know how to build bifurcation diagrams..
- \* E10: To know to characterize chaos and know how to calculate Lyapunov exponents.
- \* E11: To know how to apply dynamical systems techniques to physical, chemical, biological and social systems..

### Generic

- \* TG1: To be able to describe, both mathematically and physically, complex systems in different situations.
- \* TG2: To acquire the capacity to develop a complete research plan covering from the bibliographic research and strategy to the conclusions..
- \* TG6: To acquire high power computation skills and advanced numerical methods capabilities in applications to problems in the context of complex systems..

### Basic

- \* You may consult the basic competencies students will have to achieve by the end of the Master's degree at the following address: [http://estudis.uib.cat/master/comp\\_basiques/](http://estudis.uib.cat/master/comp_basiques/)

## Content

### Theme content

#### 1. Introduction

Phase Space, Existence and unicity of trajectories, Liouville theorem, Hamiltonian vs dissipative systems.

#### 2. One dimensional flows

Geometric representation. Fixed points. Potential representation. Stability analysis. Saddle-node bifurcation. Transcritical bifurcation. Pitchfork bifurcation. Normal forms. Bifurcation diagrams. Structural stability. Imperfect bifurcations and catastrophes.

#### 3. Two dimensional flows

Phase portraits. Fixed points. Stability. Forced damped oscillators. Limit Cycles. Index theory. Hopf bifurcation. Gradient systems. Lyapunov functions. Poincaré Bendixson theorem. Liénard Systems. Van Der Pol oscillator. Relaxation oscillations. Weakly nonlinear oscillators. Multiple time scale analysis.

#### 4. One dimensional maps. Chaos

Logistic map. Fixed points. Periodic solutions. Chaos. Calculation of Lyapunov exponents. Routes to chaos. Universality. Feigenbaum's renormalization theory.

#### 5. Three dimensional flows

Lorenz model. Chaos. Strange attractors. Poincaré map. Lorenz map. Lyapunov exponents. Floquet analysis.

#### 6. Fractals

Cantor set. Self-similarity. Dimension of self-similar fractals: Hausdorff, box counting, information and correlation dimensions. Kaplan-Yorke conjecture. Generalized dimensions  $D_q$ .

## Teaching guide

### 7. Entrainment

Circle map. 1:1 frequency locking. Rational lockings. Arnold tongues. Devil's staircase.

### 8. Synchronization of oscillators

Weakly coupled oscillators. Reduction to phase dynamics. Synchronization. Landau-Stuart oscillators. Oscillator death. Kuramoto model. Diversity. Order Parameter. Self-consistent solution.

### 9. Excitability

Biological motivation. Active rotator. Fitzhugh-Nagumo.

### 10. Non-linear time series analysis

Poincaré section. Fourier characterization. Embedding methods.

### 11. Delayed systems

Delay in physical and biological systems. Fixed points. Stability analysis. Mackey-Glass model.

## Teaching methodology

### In-class work activities

| Modality           | Name               | Typ. Grp.        | Description   | Hours |
|--------------------|--------------------|------------------|---|-------|
| Theory classes     | Theory classes     | Large group (G)  | Lectures explaining the theoretical concepts given by the professor.  | 28    |
| Practical classes  | Practical sessions | Large group (G)  | Resolution of practical examples and questions.   | 6     |
| Laboratory classes | Lab sessions       | Medium group (M) | This activity aims at the visualization of the nonlinear phenomena in real experimental systems. Experiments will be performed in mechanical, electronic or chemical systems. | 2.5   |
| Assessment         | Exam               | Large group (G)  | The student has to answer the questions of a written exam.  | 1     |

At the beginning of the semester a schedule of the subject will be made available to students through the UIBdigital platform. The schedule shall at least include the dates when the continuing assessment tests will be conducted and the hand-in dates for the assignments. In addition, the lecturer shall inform students as to whether the subject work plan will be carried out through the schedule or through another way included in the Campus Extens platform.

### Distance education work activities

| Modality              | Name   | Description  | Hours |
|-----------------------|--|--|-------|
| Individual self-study | Exercises                                    | The student has to solve exercises assigned and present the solutions in written form.                       | 50    |
| Individual self-study | Study and understanding theoretical concepts | This activity aims at the understanding of the theoretical concepts and techniques explained in the lectures | 62.5  |

## Teaching guide

### Specific risks and protective measures

The learning activities of this course do not entail specific health or safety risks for the students and therefore no special protective measures are needed.

### Student learning assessment

#### Exam

|                     |   |
|---------------------|---|
| Modality            | Assessment  |
| Technique           | Objective tests ( <b>retrievable</b> )                        |
| Description         | The student has to answer the questions of a written exam.    |
| Assessment criteria | Accuracy of the answers. Clarity in the written explanations. |

Final grade percentage: 50%

#### Exercises

|                     |  |
|---------------------|--|
| Modality            | Individual self-study  |
| Technique           | Papers and projects ( <b>retrievable</b> )   |
| Description         | The student has to solve exercises assigned and present the solutions in written form. |
| Assessment criteria | Accuracy of the answers. Clarity and quality of the explanations.                      |

Final grade percentage: 50%

### Resources, bibliography and additional documentation

#### Basic bibliography

S.H. Strogatz, "Nonlinear Dynamics and chaos", Addison Wesley 1994 / Westview Press 2000.  
E. Ott, "Chaos in Dynamical Systems", Cambridge University Press, 2nd edition, 2002.

#### Complementary bibliography

K.T. Alligood, T.D. Sauer, and J.A. Yorke, "Chaos. An Introduction to Dynamical Systems", Springer, 1997 [mainly for topics 4-5,7].  
A. Pikovsky, M. Rosenblum, J. Kurths, "Synchronization: A universal concept in nonlinear sciences", Cambridge University Press, 2001 [mainly for topics 7-8].  
S.H. Strogatz, "From Kuramoto to Crawford", Physica D vol. 143, p. 1 (2000) [mainly for topic 8].  
J.D. Murray, "Mathematical biology", 3rd edition, Springer, 2003 [mainly for topic 9].  
H.D.I. Abarbanel, "Analysis of Observed Chaotic Data", Springer, 1996 [mainly for topic 10].  
T. Erneux, "Applied Delay Differential Equations", Springer, 2009. [mainly for topic 11].